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A Model of Macroeconomic Policies with Endogenously Insurable Exchange Rates

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A Model of Macroeconomic Policies with Endogenously Insurable Exchange Rates

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ABSTRACT: We propose a tractable small-open-economy model in which uncovered interest parity premia on foreign exchange (FX) markets arise from the endogenous lack of insurability of exchange rate risks. Asymmetric information about monetary policy can make the tails of the exchange rate distribution uninsurable in FX hedging markets, which in turn generates premia in FX spot markets because of the risk aversion of lenders holding the external debt. There is a role for government intervention because of an information sensitivity externality: agents do not internalize that their actions can reduce the insurability of exchange rates. The model predicts that premia may be amplified by weaknesses in monetary, fiscal, and financial policy frameworks, and can be reduced through reforms which reduce the sensitivity of exchange rates to private information. The constrained efficient policy depends on the composition of external lenders and may include delegation of monetary policy to an “aloof central banker”, constraints on fiscal policy, more active use of macroprudential tools, and institutions which limit asymmetric information.

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1 Introduction

Many countries, especially emerging and developing ones, have foreign exchange (FX) markets characterized by significant deviations from uncovered interest parity (UIP). These deviations, i.e., UIP *premia*, mean that exchange rates become de-anchored from monetary policy rates and expectations of economic activity, while capital allocation may additionally be distorted by high external financing costs.[1] In some countries, these premia emerge and dissipate abruptly, while in other countries, they persist structurally. For policymakers, it is important to understand these premia, because the appropriate policy mix to handle them depends on why the premia arise to begin with.

In this paper, we focus on the notion that UIP premia may arise because FX market participants may not be able to insure all exchange rate risks. Four recent empirical findings may be illuminating in this regard. First, Kalemli-Özcan and Varela (2021) show that premia in emerging markets comove with policy uncertainty. Second, premia in both emerging markets and advanced economies appear to comove with a higher insurance cost for exchange rate tail risks (see Table 1, which extends the findings in Farhi and Gabaix, 2016, and Della Corte et al., 2016).[2] Third, empirical analyses of FX market microstructure point to substantial heterogeneity of participants and possibly of information (e.g., Menkhoff et al., 2016, Ranaldo and Somogyi, 2021, Cespa et al., 2022, and Hacioğlu-Hoke et al., 2024). Fourth, some countries may have deepened FX markets while increasing exchange rate volatility (e.g., Albagli et al., 2020).

Our contribution is to propose a tractable small-open-economy model in which UIP premia are related to the *endogenous insurability* of exchange rates. The tails of the exchange rate distribution may become uninsurable because of asymmetric information in the FX hedging market, and this partial uninsurability may generate premia because risks must be borne by risk-averse external holders of local currency debt in the FX spot market. The relevant asymmetric information pertains to the country's monetary policy, because this policy affects the exchange rate distribution and its tails. Such a model potentially ties together the above empirical evidence by highlighting the transmission of some kinds of policy uncertainty into asymmetric information which makes it difficult to insure exchange rate tails. In our model, the constrained efficient policy mixes and institutional designs reduce premia by modifying the sensitivity of monetary policy to private information.

[1] Evidence of UIP premia dates back to Fama (1984) and is summarized by Engel (2014) and Kalemli-Özcan and Varela (2021). The latter paper documents that emerging markets experience higher and more volatile premia than those in advanced economies. Hassan and Zhang (2021) argue that these premia reduce capital accumulation.

[2] This table correlates UIP premia calculated using the method in Das et al. (2022) against a J.P. Morgan series that is related to one previously introduced in Farhi et al. (2015) and Farhi and Gabaix (2016), but covers emerging markets as well. We include countries with predominantly non-stabilized exchange rate regimes.

Table 1: Regression of UIP Premium (bps) on Downside TRPP (z-scored)

Specification	(1)	(2)
Coefficient	139.40***	154.59***
Standard error	(38.45)	(37.52)
R-squared	0.017	0.140
Observations	5519	5519
Country fixed effects	No	Yes

Note: UIP premium calculation follows Das et al. (2022) using data from Refinitiv, the central banks of Brazil and Chile, and Consensus Forecasts. TRPP is the tail risk protection premium from the J. P. Morgan JPMaQS Database, i.e., the average ratio of Black-Scholes implied volatility of out-of-the-money 10-delta options relative to at-the-money 50-delta options. Time period: Monthly series between Jan 2006 and Dec 2024. Countries: 15 emerging markets (Brazil, Chile, Hungary, India, Indonesia, Malaysia, Mexico, Peru, Philippines, Poland, Romania, Russia, South Africa, Thailand, and Turkey) and 12 advanced economies (Australia, Canada, Czech Republic, Euro area, Israel, Japan, Korea, New Zealand, Norway, Sweden, Switzerland, and UK). Standard errors: Two-way clustered for country and time. Significance level: *** indicates a p-value < 0.01.

In the first part of the paper, we propose a general model which combines the portfolio balance approach to FX markets with an information asymmetry inspired by the empirics of FX market microstructure. The model has two periods and two kinds of shocks. Monetary policy amounts to selecting the policy rate and exchange rates. Premia are determined by the interaction of FX hedging and spot markets in the first period in anticipation of shocks. The FX hedging market comprises uninformed insurers and informed speculators who interact in a market with sticky insurance prices, while the FX spot market includes risk-averse external lenders who hold the local currency debt issued by domestic agents and insure the exchange rate volatility using FX hedges. In the second period, the shocks are realized and observable to all agents. The exchange rate is restricted to be a linear function of both shocks, but there is a key difference between them: in the first period, the *public* shock is not known to anyone, but the *private* shock is known to the informed speculators.

Premia are generated in our framework as follows. First, the FX hedging market breaks down for a particular exchange rate if uninformed insurers and informed speculators cannot agree on its probability. This problem occurs only if monetary policy sets exchange rates to vary with the private shock, and then only for exchange rates in the tails of the distribution. Exchange rates in the middle of the distribution can plausibly be generated by some combination of public and private shocks, so the insurers and speculators can have different information sets but still agree on the probabilities of the exchange rates. However, exchange rates in the tails only occur when the private shock is especially small or large, so the speculators know the probabilities of those exchange rates more accurately than the insurers do. The insurers risk large losses when trading with the speculators, so they will offer no contracts to insure such exchange rates.[3] Second, UIP premia can then arise on the FX spot market because if risk-averse external lenders cannot insure the tails of the exchange rate distribution, they charge a premium for bearing the risk of these tails themselves.

Our model suggests that premia arise structurally in countries whose monetary policy is sensitive to shocks known beforehand only to a subset of FX hedging market participants, and may exhibit episodic surges when such shocks become more important or there is a risk of a regime shift that makes monetary policy more sensitive to such shocks. This insight may help explain the empirical connection between UIP premia and measures of policy uncertainty. It may also explain why the premia are especially large in emerging and developing countries, as they may be more susceptible to asymmetric information than advanced countries.

Since insurability is endogenous to macroeconomic policies in our model, those policies should be altered to improve insurability and reduce premia. In our environment, there is an *information sensitivity externality*: when agents respond to the private shock in the second period, they do not internalize that their actions also cause the exchange rate to respond to that shock in that period, which in turn generates premia in the first period. From the planner's perspective, it may be desirable to reduce the relative sensitivity of the exchange rate to the private shock vis-à-vis the public shock. If so, the planner would need to commit in the first period to a monetary policy such that the exchange rate in the second period varies less with the private shock. There may be a case for delegating monetary policy to an "aloof central banker" who has the ability to down-weight that shock in the second period.[4]

Our focus on endogenous insurability in the FX hedging market builds on other recent contributions in the portfolio balance literature in international finance.[5] Previous work has explored the relationship between UIP premia and exchange rate volatility (see Gabaix and Maggiori, 2015, and Itskhoki and Mukhin, 2021, 2023). Chernov et al. (2024) examine how premia can be altered when agents hedge risks using financial assets from outside the FX market. By adding the notion of endogenous insurability, we show that premia depend differently on different kinds of exchange rate volatility: they depend specifically on the volatility of exchange rates with respect to shocks over which there is asymmetric information, while other forms of volatility can be insured away.

[3] We incorporate the suggestion in some of the microstructure literature that more informed agents make profits off less informed ones, and for the sake of tractability we impose a sticky-price assumption that amplifies profits and losses.

[4] This case for monetary delegation is distinct from the inflation bias rationale in Rogoff (1985).

[5] The portfolio balance approach dates back to Kouri (1976) and includes Cavallino (2019), Amador et al. (2020), Fanelli and Straub (2021), and Basu et al. (2025) as well as the papers listed in this paragraph.

Our model focuses on a small open economy and FX hedging markets which yield full insurability under symmetric information. Moving beyond our model to a global economy context, global insurers should remain able to fully absorb exchange rate risks away from external lenders at actuarially fair prices for idiosyncratic country-specific shocks. This outcome is not possible in the models of Farhi and Gabaix (2016) and Hassan et al. (2023) for undiversifiable global risks and/or large countries, so premia can arise there even with symmetric information and complete markets.

Our monetary policy result evokes and yet contrasts with a separate literature on information sensitivity. Dang et al. (2015, 2017) examine how private institutions can design securities to make them less sensitive to private information and thereby more liquidly tradable by uninformed agents. By contrast, our framework takes as given that domestic agents issue local currency debt securities, and then it explores how macroeconomic policies affect the sensitivity of the FX returns on that debt to private information: the less sensitive they are, the lower the premia.

We model UIP premia and FX hedging markets, but not the specifics of the FX forward market where covered interest parity (CIP) premia may arise.[6] These CIP premia are related to the constraints faced by the traders of forward contracts. In the cases where our model produces full insurability in the FX hedging market, those traders can potentially use FX hedges to replicate a forward contract, which may affect CIP premia and the forward exchange rate.

In the second part of the paper, we illustrate the adaptability of the general model by exploring four applications of it which can shed light on different kinds of policies.

The first application links the magnitude of premia to the composition of external lenders and the level of external debt. We explore two options. One option is that these lenders are owned by foreigners with log utility. If so, FX market premia become large if there are insurability problems and each lender's exposure to the country's external debt is large—which can arise if the set of lenders who specialize in the country's debt is small and/or the debt becomes large. The premia also varies with how much of the debt is held by inframarginal investors (e.g., the "noise traders" in Gabaix and Maggiori, 2015). Another option is that the lenders are owned by foreigners who face financial constraints after large losses. If so, premia may be driven mostly by the risk of large depreciations.

The second application explores the influence of fiscal policy on FX market premia. We include public spending on imports, with spending in the second period being affected by the private shock. Fiscal policy then affect premia via two channels. First, public spending in the first period increases external debt and thereby the premia on that debt. Second, the dependence of public spending on the

[6]Du and Schreger (2022) catalogue CIP premia and its drivers, while Dao et al. (2025) provide a framework to reconcile UIP and CIP premia.

private shock generates macroeconomic volatility in the second period, which needs to be addressed via monetary policy and thereby underlies the dependence of exchange rates on the private shock described above. In our model, premia can be mitigated by cutting public spending in the first period and reducing its sensitivity to the private shock in the second period.[7]

The third application explores how financial policies could affect FX market premia. We assume that the country's output in the second period depends on a financial shock which is known in advance only by the informed speculators. We add the possibility that the planner can build an FX fund in the first period with which to offer liquidity injections in the second period. The planner would use such a fund even with symmetric information for macro-prudential reasons. In the presence of asymmetric information, the fund and the injections could be larger, because exchange rates can then be made less sensitive to the private financial shock and premia can thereby be reduced.[8]

The fourth application considers whether institutional design can affect FX market premia by limiting asymmetric information. We assume that the planner has three regime options: do nothing, or keep information about the private shock secret from all agents, or instead collect and publish all information related to that shock.[9] The first regime achieves the previously-discussed outcome with partial FX hedging market breakdown and positive premia. The other two regimes of secrecy and publication both achieve symmetric information in the first period, eliminating FX market premia. Our model suggests that the benefit of lower premia, which is salient when external debt is high, should be weighed against the costs of implementing these regimes.

Taken together, these four applications highlight that in our model, the development of FX markets and the behavior of premia depend on the country's use of policy instruments before and after shocks and the institutions that govern such use.

Next, we turn to the model-building and analysis. Section 2 lays out the general model. Section 3 extracts some over-arching results from this model. Section 4 then covers the four applications described above. Section 5 concludes. All the proofs of our results are in the appendix.

[7]This effect of fiscal policy on insurability constitutes another reason for disciplining fiscal policy beyond those in the literature on business cycle management and sovereign debt risks. In that literature, fiscal rules and institutions can help reduce public debt (e.g., Wyplosz, 2013; Halac and Yared, 2014; Hatchondo et al., 2022).

[8]This reasoning suggests that relative to recent models on the integrated use of financial policy tools alongside monetary policy (e.g., Farhi and Werning, 2016; Bianchi and Lorenzoni, 2022; Basu et al., 2025), the tools may have additional benefits (or costs) depending on whether they decrease (or increase) the sensitivity of monetary policy to private information.

[9]We assume that the public and private shocks correspond to distinct underlying macroeconomic variables. This modeling is different from that in papers which consider public and private signals about the same underlying variable (e.g., global games as analyzed by Hellwig, 2002). However, in the language of these other papers, our option to collect and publish the private information can be interpreted as the production of a costly, zero-variance, and publicly-observable signal about the private shock which renders redundant the existing private signal about that shock.

2 General Model

In this section, we introduce our stylized small open economy with flexible exchange rates, local currency debt, asset market segmentation, and asymmetric information in the FX market. It includes households, home goods firms, external lenders, insurers, and speculators.

2.1 Environment

Shocks and information structures. There are two periods $t \in \{1, 2\}$ and two shocks: a public shock φ and a private shock θ which each take values within $[0, 1]$. These shocks are realized and observable to all agents at $t = 2$, and together they characterize the economy's state of nature (φ, θ) in that period. All variables with the subscript $t = 2$ are actually functions of this state of nature. The shocks are not contractible, so no hedging contracts exist to directly insure them at $t = 1$.[10]

At $t = 1$, all agents may have one of two information structures (i.e., sigma-algebras): the uninformed structure $\mathcal{I}_\emptyset$ or the informed structure $\mathcal{I}_\theta$. From the perspective of the uninformed agents, no shocks are known, and the two shocks follow independent uniform distributions over $[0, 1]$. From the perspective of the informed agents, the public shock follows a uniform distribution over $[0, 1]$ but these agents know the value of the private shock θ. $\pi(\theta|\mathcal{I})$ is the probability density function of the private shock θ given the information structure $\mathcal{I}$, so $\pi(\theta|\mathcal{I}_\emptyset) = 1$ for all $\theta \in [0, 1]$ while $\pi(\theta|\mathcal{I}_\theta) = 1$ only for the known value of θ and $= 0$ for other values (strictly speaking, there is a mass point at θ for the latter information structure).

For the lenders, insurers, and speculators, we consider different configurations of their respective information structures $\mathcal{I}_L$, $\mathcal{I}_I$, and $\mathcal{I}_S$ at $t = 1$. Each of these structures is equal to one of $\{\mathcal{I}_\emptyset, \mathcal{I}_\theta\}$ and we assume that $\mathcal{I}_L = \mathcal{I}_I \subseteq \mathcal{I}_S$, i.e., the speculators are at least as informed as the lenders and insurers. All other agents in the model are always uninformed, so we omit the notation for their information structures. There is no information revelation within $t = 1$, so agents' information structures do not get updated in this period.

Monetary policy. The planner's monetary policy amounts to setting the policy rate i_1 between the two periods and the exchange rates $\{\mathcal{E}_1, \mathcal{E}_2\}$ in each period (in units of local currency per dollar), i.e., a single number at $t = 1$ and a function of (φ, θ) at $t = 2$. In this paper, we restrict the planner to a linear function $\mathcal{E}_2 = \alpha\varphi + \beta\theta + \gamma$. We define Ψ as the set of possible exchange rates $\mathcal{E}_2$ at $t = 2$ from the perspective of the lenders and insurers at $t = 1$. $\pi(\mathcal{E}_2|\mathcal{I})$ is the probability density

[10] Section 4 explores some examples of public and private shocks. We exclude shocks for which direct hedging contracts exist, e.g., certain commodity prices, as such shocks can be insured without relying on FX hedging markets.

function of the exchange rate $\mathcal{E}_2$ given the information structure $\mathcal{I}$ at $t = 1$. Correspondingly, $\pi(\mathcal{E}_2|\mathcal{I}_\emptyset) = \int_{\theta=0}^1 \pi(\mathcal{E}_2|\mathcal{I}_\theta)\, d\theta$, where $\mathcal{I}_\theta$ is evaluated at each θ. The expectation operator for any variable x can be written in two equivalent forms:

$$\mathbb{E}[x|\mathcal{I}] = \int_{\theta=0}^1 \int_{\varphi=0}^1 x\pi(\theta|\mathcal{I})\, d\varphi d\theta = \int_{\mathcal{E}_2 \in \Psi} x\pi(\mathcal{E}_2|\mathcal{I})\, d\mathcal{E}_2.$$

The exchange rate is contractible, so FX hedging contracts may exist. We denote $\widetilde{\Psi}$ as the insurable set, i.e., the set of exchange rates for which such contracts exist, where $\widetilde{\Psi} \subseteq \Psi$.

Consumption and production. This part of the environment is adapted from Farhi and Werning (2016). A continuum measure one of households maximize their welfare:

$$\max_{\{C_{Ht}, C_{Ft}, N_t\}} \mathbb{E}\left[\sum_{t=1}^{2} \delta^{t-1} U(C_{Ht}, C_{Ft}, N_t)\right]$$

where $U(C_{Ht}, C_{Ft}, N_t) = \sigma_H \log C_{Ht} + \sigma_F \log C_{Ft} - N_t$ and $\sigma_F = 1 - \sigma_H$,

subject to the budget constraint:

$$W_t N_t + \Pi_{Ht} + \mathcal{E}_t Y_{Ft} + T_t + D_t \geq P_H C_{Ht} + \mathcal{E}_t C_{Ft} + (1 + i_{t-1}) D_{t-1}.$$

On the income side of the constraint, W_t is the wage, N_t is labor supply, Π_{Ht} is the profit of home goods firms, Y_{Ft} is the endowment of tradable goods, T_t is the lump-sum transfer from the planner, and D_t is the local currency debt at the end of period t. On the expenditure side, P_H and C_{Ht} are the local price (fully rigid over time) and consumption of home goods, while C_{Ft} is the consumption of imports. We normalize the dollar prices of tradable endowments and imports to 1 and i_0 to 0. Debt can be non-zero to begin with but is extinguished at the end of time, i.e., $D_2 = 0$. Households' tradable endowments Y_{Ft} are a function of (φ, θ) at $t = 2$, but they have no access to insurance contracts.

Their first order conditions (FOCs) yield the following intratemporal and Euler conditions:

$$C_{Ht} = \frac{\sigma_H}{\sigma_F} \frac{\mathcal{E}_t}{P_H} C_{Ft},\, W_t = \frac{1}{\sigma_F} \mathcal{E}_t C_{Ft}, \text{ and } \frac{1}{\mathcal{E}_1 C_{F1}} = \delta(1 + i_1)\, \mathbb{E}\left[\frac{1}{\mathcal{E}_2 C_{F2}}\right].$$

A continuum measure one of home goods firms are monopolistically competitive and set prices at the beginning of period $t = 1$, after which prices are fully rigid. Each firm produces a variety $j \in [0, 1]$ of the home good using the production technology $Y_{Ht}(j) = A_t N_t(j)$, where productivity is exogenously given by A_t and its value is a function of (φ, θ) at $t = 2$. The home good aggregator, the variety-level demand function, and the aggregate price index are:

$$Y_{Ht}=\left(\int_0^1 Y_{Ht}\left(j\right)^{\frac{\varepsilon-1}{\varepsilon}}dj\right)^{\frac{\varepsilon}{\varepsilon-1}},Y_{Ht}\left(j\right)=Y_{Ht}\left(\frac{P_H\left(j\right)}{P_H}\right)^{-\varepsilon},\text{ and }P_H=\left(\int_0^1 P_H\left(j\right)^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}},$$

where $\varepsilon>1$. Labor is taxed by the planner at the pre-set rate ϕ. The variety is priced at $P_H\left(j\right)$ in local currency, and this price is set to maximize expected profits:

$$\max_{P_H(j)}\mathbb{E}\left[\sum_{t=1}^{2}s_t\left[P_H\left(j\right)Y_{Ht}\left(j\right)-\left(1+\phi\right)W_tN_t\left(j\right)\right]\right],$$

where $s_t\equiv\delta^{t-1}\frac{\mathcal{E}_1C_{F1}}{\mathcal{E}_tC_{Ft}}$ is the households' period-1 stochastic discount factor (SDF) for local currency income in period t (taken by firms as an exogenous function of time and states of nature (φ,θ)). The FOC for price-setting establishes the following expression:

$$P_H\left(j\right)=\left(1+\phi\right)\frac{\varepsilon}{\varepsilon-1}\frac{\mathbb{E}\left[\sum_{t=1}^{2}\delta^{t-1}\frac{1}{\mathcal{E}_tC_{Ft}}\frac{W_t}{A_t}Y_{Ht}\right]}{\mathbb{E}\left[\sum_{t=1}^{2}\delta^{t-1}\frac{1}{\mathcal{E}_tC_{Ft}}Y_{Ht}\right]}.$$

Since all firms are identical, $P_H=P_H\left(j\right)$, $Y_{Ht}=Y_{Ht}\left(j\right)$, and $N_t=N_t\left(j\right)$.

The labor market clears at the flexible wage W_t, and there is market clearing for home goods, i.e., $Y_{Ht}=C_{Ht}$.

FX spot market. This market is open at $t=1$. It includes the above households who issue local currency debt and a continuum measure one of external lenders who absorb that debt. External lenders can undertake two kinds of financial transactions at $t=1$. First, they can exchange currencies by taking a position of D_1 in local currency bonds financed by a position of $-\frac{D_1}{\mathcal{E}_1}$ in dollar bonds. Second, they have access to the available hedging contracts over exchange rates. For each exchange rate $\mathcal{E}_2$ that lies within the insurable set $\widetilde{\Psi}$, the lenders can purchase a quantity $Q_L\left(\mathcal{E}_2\right)$ of contracts at price $p\left(\mathcal{E}_2\right)$ which each yield one dollar if that exchange rate materializes at $t=2$. The lenders maximize the expected value of their dollar profits for their foreign owners:

$$\max_{\{B_1,Q_L(\mathcal{E}_2)\}}\mathbb{E}\left[M_2\left(\left[\eta_2-\left(1+i^*\right)\right]B_1+Q_L\left(\mathcal{E}_2\right)\right)|\mathcal{I}_L\right]$$

$$\text{where }\eta_2\equiv\left(1-\mu_1\right)\left(1+i_1\right)\frac{\mathcal{E}_1}{\mathcal{E}_2}\text{ and }B_1\equiv\frac{D_1}{\mathcal{E}_1},$$

subject to the constraint of zero net purchases of the available insurance contracts:

$$\int_{\mathcal{E}_2\in\widetilde{\Psi}}p\left(\mathcal{E}_2\right)Q_L\left(\mathcal{E}_2\right)d\mathcal{E}_2=0.$$

M_2 is the foreigners' period-1 SDF over dollar income at $t = 2$ (taken by lenders as an exogenous function), η_2 is the dollar value of the return on local currency bonds for the external lenders, μ_1 is a tax on capital inflows set by the planner at $t = 1$, i^* is the foreign interest rate, and B_1 is the dollar value of the local currency bondholdings. The lenders' FOCs for the bond position and the quantity of insurance contracts are respectively as follows:

$$\mathbb{E}\left[M_2\left[\eta_2-(1+i^*)\right]|\mathcal{I}_L\right]=0 \text{ and } Q_L\left(\mathcal{E}_2\right)\begin{cases}\to\infty & \text{if } M_2>\overline{M}\frac{(1+i^*)p(\mathcal{E}_2)}{\pi(\mathcal{E}_2|\mathcal{I}_L)}\\ \in\mathbb{R} & \text{if } M_2=\overline{M}\frac{(1+i^*)p(\mathcal{E}_2)}{\pi(\mathcal{E}_2|\mathcal{I}_L)}\\ \to-\infty & \text{if } M_2<\overline{M}\frac{(1+i^*)p(\mathcal{E}_2)}{\pi(\mathcal{E}_2|\mathcal{I}_L)},\end{cases}$$

where $\overline{M}$ is a positive constant.

While M_2 is not affected by the decisions of each lender, it is affected by the *aggregate* decisions of the set of lenders as well as other relevant equilibrium variables. We assume that the SDF takes the form of a continuously differentiable function $M_2 = M_2\left(B_1, \eta_2, Q_L\left(\mathcal{E}_2\right)\right)$ defined over the lenders' aggregate decisions and the dollar return, with the following properties at each $\mathcal{E}_2$:

$$\frac{dM_2}{dQ_L\left(\mathcal{E}_2\right)}<0,\ \lim_{Q_L(\mathcal{E}_2)\to\infty}M_2=0,\text{ and }\lim_{Q_L(\mathcal{E}_2)\to(Q_{\min}(\mathcal{E}_2))^+}M_2=\infty\text{ for some }Q_{\min}\left(\mathcal{E}_2\right).$$[11]

Combining these conditions with the lenders' FOC for the insurance contracts, we derive the following condition which produces a finite $Q_L\left(\mathcal{E}_2\right)$ for each $\mathcal{E}_2$:

$$M_2=\overline{M}\frac{\left(1+i^*\right)p\left(\mathcal{E}_2\right)}{\pi\left(\mathcal{E}_2|\mathcal{I}_L\right)}\text{ for all }\mathcal{E}_2\in\widetilde{\Psi}.$$

The solution to the FX spot market yields the UIP premium $\rho \equiv \mathbb{E}\left[\eta_2\right] - (1 + i^*)$.

FX hedging markets. These markets are open to a set of foreign agents at $t = 1$. They include a continuum measure one of risk-neutral insurers and speculators alongside the external lenders described above. There is a continuum of markets, one for each exchange rate $\mathcal{E}_2 \in \Psi$, and all the variables within each market are indexed by $\mathcal{E}_2$.[12] The density of the insurers and speculators at each exchange rate is $\frac{1}{\psi}$, where ψ is the width of the support of $\mathcal{E}_2$ from the insurers' perspective, i.e., $\psi = (\alpha + \beta)$ if $\mathcal{I}_I = \mathcal{I}_\emptyset$ and $\psi = \alpha$ if $\mathcal{I}_I = \mathcal{I}_\theta$. Each market operates with the following sequence within $t = 1$:

[11]These properties are satisfied by the classes of foreigners' welfare maximization examined in subsection 4.1. The assumed M_2 function does not depend on the shocks (φ, θ) beyond their impact on $\{B_1, \eta_2, Q_L\left(\mathcal{E}_2\right)\}$.

[12]In our notation, the price and quantity variables are indexed by $\mathcal{E}_2$ through analogy with the above analysis of the continuum of goods varieties indexed by j. As in that analysis, this notation does *not* mean that prices and quantities depend only on $\mathcal{E}_2$. In the FX hedging markets, the quantities chosen by the speculators and lenders depend on their own information structures as well as on the prices offered by the insurers.

- Step 1. The insurers for this exchange rate first decide their participation, $d \in \{0, 1\}$. If $d = 1$, they set a price $p(\mathcal{E}_2)$ and commit to sell the quantity $Q_I(\mathcal{E}_2)$ of contracts needed to fulfill all demands. They maximize their expected profits:

$$\max_{p(\mathcal{E}_2)} \int_{\theta=0}^{1} \left[(1+i^*)\, p(\mathcal{E}_2)\, Q_I(\mathcal{E}_2) - \pi(\mathcal{E}_2|\mathcal{I}_S)\, Q_I(\mathcal{E}_2)\right] \pi(\theta|\mathcal{I}_I)\, d\theta.$$

  The first term inside the square bracket represents the insurers' revenues at $t = 1$, compounded with interest, while the second term represents their expected payouts at $t = 2$. The integral over θ reflects that the insurer may be uncertain as to what quantities the speculators will demand in step 2, to the extent that the latter have a superior information set $\mathcal{I}_S \supset \mathcal{I}_I$. If the above expected profits are negative, the insurers opt instead for $d = 0$, in which case they offer no price, do not fulfill any contracts, and earn zero profits.

- Step 2. If $d = 1$, speculators and lenders observe $p(\mathcal{E}_2)$ and set their demands without observing each other's demands. Speculators purchase a quantity $Q_S(\mathcal{E}_2)$ of contracts to maximize their expected profits:

$$\max_{Q_S(\mathcal{E}_2)} \left[\pi(\mathcal{E}_2|\mathcal{I}_S)\, Q_S(\mathcal{E}_2) - (1+i^*)\, p(\mathcal{E}_2)\, Q_S(\mathcal{E}_2)\right].$$

  The first term of the maximand represents the speculators' expected revenues at $t = 2$, while the second term represents their expenses at $t = 1$, compounded with interest. Lenders purchase a quantity $\psi Q_L(\mathcal{E}_2)$ of contracts derived from the FX spot market analysis above, where the term ψ adjusts for their measure relative to that of the insurers and speculators. If $d = 0$, speculators and lenders cannot demand any contracts.

- Step 3. The market for insurance contracts clears, pinning down the quantity of contracts that the insurer must fulfill, i.e., $Q_I(\mathcal{E}_2) = Q_S(\mathcal{E}_2) + \psi Q_L(\mathcal{E}_2)$. If $d = 1$, the exchange rate $\mathcal{E}_2$ is insurable, i.e., $\mathcal{E}_2 \in \widetilde{\Psi}$. If $d = 0$, the exchange rate $\mathcal{E}_2$ is not insurable, so $p(\mathcal{E}_2) = \emptyset$ and $Q_I(\mathcal{E}_2) = Q_S(\mathcal{E}_2) = Q_L(\mathcal{E}_2) = 0$.

The outcome of this problem depends on the information structures of the insurer and speculators, and it determines the insurable set of exchange rates $\widetilde{\Psi}$. We solve it in the next subsection.

Lump sum rebates. The planner rebates to households in lump sum the revenues from taxes on labor and capital inflows:

$$T_t = \phi W_t N_t + \mu_{t-1}(1+i_{t-1})\, D_{t-1}.$$

The competitive equilibrium is defined as follows.

Definition 1 (Competitive equilibrium). *A competitive equilibrium comprises the quantities* $\left\{\{C_{Ht}, C_{Ft}, N_t, Y_{Ht}\}_{t=1}^{2}, D_1, \{Q_L(\mathcal{E}_2), Q_I(\mathcal{E}_2), Q_S(\mathcal{E}_2)\}_{\mathcal{E}_2\in\Psi}\right\}$*, prices* $\left\{P_H, \{W_t\}_{t=1}^{2}, \{p(\mathcal{E}_2)\}_{\mathcal{E}_2\in\Psi}\right\}$*, exchange rate sets* $\left\{\Psi, \widetilde{\Psi}\right\}$*, and probability distributions* $\{\pi(\mathcal{E}_2|\mathcal{I}_I), \pi(\mathcal{E}_2|\mathcal{I}_S)\}_{\mathcal{E}_2\in\Psi}$ *that satisfy the optimization conditions and constraints of households, home goods firms, external lenders, insurers, and speculators, as well as the market clearing conditions for home goods, labor, local currency bonds, and FX hedging contracts, given the foreigners' SDF function* M_2*, the information structures* $\{\mathcal{I}_L, \mathcal{I}_I, \mathcal{I}_S\}$*, and the values of the policy instruments* $\left\{i_1, \{\mathcal{E}_t\}_{t=1}^{2}, \phi, \mu_1\right\}$*, where* $\mathcal{E}_2 = \alpha\varphi + \beta\theta + \gamma$ *for some* $\{\alpha, \beta, \gamma\}$*.*

2.2 Insurability and Premia

The FX hedging market produces the following condition that must be satisfied for any exchange rate within the insurable set $\widetilde{\Psi}$.

Proposition 1 (Insurability condition). $\mathcal{E}_2 \in \widetilde{\Psi}$ *if and only if* $\pi(\mathcal{E}_2|\mathcal{I}_I) = \pi(\mathcal{E}_2|\mathcal{I}_S)$ *irrespective of* θ*. Within that set,* $(1+i^*)\,p(\mathcal{E}_2) = \pi(\mathcal{E}_2|\mathcal{I}_I)$*.*

This proposition establishes that for an exchange rate to be insurable, insurers and speculators need to agree on its probability—whether they have the same information structure or not. The reason is that with the given price stickiness and market structure, the insurers make large losses if speculators bet against them. The insurers can only avoid such betting if the speculators agree with the insurers on the probabilities.[13]

We can establish the proposition by solving the FX hedging market backward. Here, alongside the general mathematical steps, we provide the argument assuming that $\pi(\mathcal{E}_2|\mathcal{I}_I) > 0$ and $p(\mathcal{E}_2) > 0$; the proof in the appendix relaxes these inequalities. The proof of the proposition does not rely on $\mathcal{E}_2$ being a linear function of (φ, θ).

In step 2, speculators have the demands:

$$Q_S(\mathcal{E}_2)\begin{cases} \to\infty & \text{if } \pi(\mathcal{E}_2|\mathcal{I}_S) > (1+i^*)\,p(\mathcal{E}_2) \\ \in\mathbb{R} & \text{if } \pi(\mathcal{E}_2|\mathcal{I}_S) = (1+i^*)\,p(\mathcal{E}_2) \\ \to-\infty & \text{if } \pi(\mathcal{E}_2|\mathcal{I}_S) < (1+i^*)\,p(\mathcal{E}_2)\,. \end{cases}$$

Since the external lenders are risk averse, $Q_L(\mathcal{E}_2)$ is finite. Correspondingly, the speculators' demand for insurance determines the sign of $Q_I(\mathcal{E}_2)$.

[13] This result is not akin to full revelation of information (indeed, no information is revealed within $t = 1$). Instead, it applies given the information structures that insurers and speculators begin with, even if they are different.

In step 1, the insurers' problem depends on the information structures. They know that each value of the exchange rate $\mathcal{E}_2$ can be the result of a combination of the public shock φ and the private shock θ. There are two cases, which we next analyze in turn.

The first case is that $\mathcal{I}_I = \mathcal{I}_S \in \{\mathcal{I}_\emptyset, \mathcal{I}_\theta\}$, i.e., the insurers and speculators are either both uninformed or both informed, irrespective of the specific value of θ. If the insurers choose $d = 1$, their expected profits are as follows:

$$\max_{p(\mathcal{E}_2)} \left[(1+i^*)\, p(\mathcal{E}_2) - \pi(\mathcal{E}_2|\mathcal{I}_I)\right] \left[Q_S(\mathcal{E}_2) + \psi Q_L(\mathcal{E}_2)\right].$$

The insurers are willing to select $d = 1$ and earn zero profits by setting $p(\mathcal{E}_2)$ such that $(1+i^*)\, p(\mathcal{E}_2) = \pi(\mathcal{E}_2|\mathcal{I}_I) = \pi(\mathcal{E}_2|\mathcal{I}_S)$. At any other price, they would make losses.

The second case is that $\mathcal{I}_F = \mathcal{I}_\emptyset$ and $\mathcal{I}_S = \mathcal{I}_\theta$, i.e., the insurers are uninformed and speculators are informed. If the insurers choose $d = 1$, their expected profits are as follows:

$$\begin{aligned}
&\max_{p(\mathcal{E}_2)} \int_{\theta:\, \pi(\mathcal{E}_2|\mathcal{I}_S) > (1+i^*)p(\mathcal{E}_2)} \left\{ \begin{array}{c} \left[(1+i^*)\, p(\mathcal{E}_2) - \pi(\mathcal{E}_2|\mathcal{I}_S)\right] \\ \times \left[Q_S(\mathcal{E}_2) + \psi Q_L(\mathcal{E}_2)\right] \end{array} \right\} d\theta \\
&+ \int_{\theta:\, \pi(\mathcal{E}_2|\mathcal{I}_S) = (1+i^*)p(\mathcal{E}_2)} \left\{ \begin{array}{c} \left[(1+i^*)\, p(\mathcal{E}_2) - \pi(\mathcal{E}_2|\mathcal{I}_S)\right] \\ \times \left[Q_S(\mathcal{E}_2) + \psi Q_L(\mathcal{E}_2)\right] \end{array} \right\} d\theta \\
&+ \int_{\theta:\, \pi(\mathcal{E}_2|\mathcal{I}_S) < (1+i^*)p(\mathcal{E}_2)} \left\{ \begin{array}{c} \left[(1+i^*)\, p(\mathcal{E}_2) - \pi(\mathcal{E}_2|\mathcal{I}_S)\right] \\ \times \left[Q_S(\mathcal{E}_2) + \psi Q_L(\mathcal{E}_2)\right] \end{array} \right\} d\theta.
\end{aligned}$$

The insurers do not know θ when they set the price $p(\mathcal{E}_2)$ in step 1, but they know that θ can lie in one of the three different regions indicated by the above integral. If θ takes a value such that $\pi(\mathcal{E}_2|\mathcal{I}_S) > (1+i^*)\, p(\mathcal{E}_2)$, the insurers make losses. If θ takes a value such that $\pi(\mathcal{E}_2|\mathcal{I}_S) = (1+i^*)\, p(\mathcal{E}_2)$, the insurers make zero profits. If θ takes a value such that $\pi(\mathcal{E}_2|\mathcal{I}_S) < (1+i^*)\, p(\mathcal{E}_2)$, the insurers again make losses.

The upshot is that the insurers select $d = 1$ if and only if they can set $p(\mathcal{E}_2)$ such that the condition $\pi(\mathcal{E}_2|\mathcal{I}_S) = (1+i^*)\, p(\mathcal{E}_2)$ holds with probability one. Since they do not know θ, they cannot set $p(\mathcal{E}_2)$ in such a manner if $\pi(\mathcal{E}_2|\mathcal{I}_S)$ varies with θ; instead, they will simply set $d = 0$. However, if $\pi(\mathcal{E}_2|\mathcal{I}_S)$ does not vary with θ, we obtain $\pi(\mathcal{E}_2|\mathcal{I}_I) = \pi(\mathcal{E}_2|\mathcal{I}_S)$ irrespective of θ, and the insurers can set $p(\mathcal{E}_2)$ to the desired value. In this case, the insurers and speculators agree on the probability of this specific exchange rate value $\mathcal{E}_2$ despite having different information structures. In other words, the probability of the range of (φ, θ) that can jointly yield $\mathcal{E}_2$ happens to be equal to the probability of the set of φ that can yield $\mathcal{E}_2$, given any particular value of θ.

Next, we turn to the insurable set and premia associated with the linear exchange rate function $\mathcal{E}_2 = \alpha\varphi + \beta\theta + \gamma$.

Proposition 2 (Insurability and premia). *Suppose that $\alpha > \beta \geq 0$. The external lenders' bond position satisfies:*

$$0 = \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\eta_2 - (1+i^*)\right] \pi\left(\theta|\mathcal{I}_I\right) d\varphi d\theta$$
$$+ \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} m_2 \left[\eta_2 - (1+i^*)\right] \pi\left(\theta|\mathcal{I}_I\right) d\varphi d\theta$$
$$+ \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} m_2 \left[\eta_2 - (1+i^*)\right] \pi\left(\theta|\mathcal{I}_I\right) d\varphi d\theta,$$

where we define $m_2 \equiv \frac{M_2}{M}$. We can establish the following cases:

(a) Full insurability: $\mathcal{I}_I = \mathcal{I}_S \in \{\mathcal{I}_\emptyset, \mathcal{I}_\theta\}$ and/or $\beta = 0$. $\widetilde{\Psi} = \Psi$, so the probability of $\mathcal{E}_2 \in \widetilde{\Psi}$ is 1 and the premium is $\rho = 0$.

(i) If $\mathcal{I}_I = \mathcal{I}_S = \mathcal{I}_\emptyset$, the insurable set is $\widetilde{\Psi} = [\gamma, \alpha + \beta + \gamma]$, while $m_2 = 1$ and $\pi\left(\theta|\mathcal{I}_I\right) = 1$.

(ii) If $\mathcal{I}_I = \mathcal{I}_S = \mathcal{I}_\theta$, the insurable set is $\widetilde{\Psi} = [\beta\theta + \gamma, \alpha + \beta\theta + \gamma]$, while $m_2 = 1$ and the integrals over θ are removed because $\pi\left(\theta|\mathcal{I}_I\right) = 1$ only at the known value of θ and $= 0$ elsewhere.

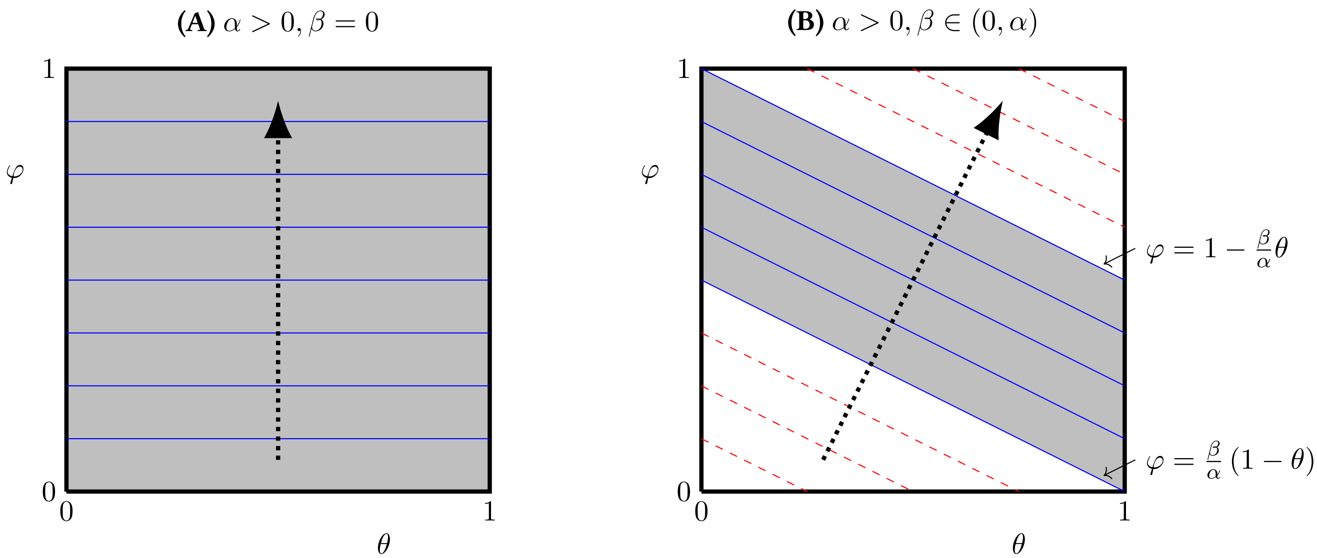
(iii) If $\mathcal{I}_I = \mathcal{I}_\emptyset$, $\mathcal{I}_S = \mathcal{I}_\theta$, and $\beta = 0$, the insurable set is $\widetilde{\Psi} = [\gamma, \alpha + \gamma]$, while $\pi\left(\theta|\mathcal{I}_I\right) = 1$ and the second and third terms of the above expression are zero.

(b) Partial insurability: $\mathcal{I}_I = \mathcal{I}_\emptyset$, $\mathcal{I}_S = \mathcal{I}_\theta$, and $\beta > 0$. The exchange rate set is $\Psi = [\gamma, \alpha + \beta + \gamma]$ and the insurable set is $\widetilde{\Psi} = [\beta + \gamma, \alpha + \gamma]$, while $\pi\left(\theta|\mathcal{I}_I\right) = 1$. The probability of $\mathcal{E}_2 \in \widetilde{\Psi}$ is $1 - \frac{\beta}{\alpha}$. The upper and lower insurability contours are $\varphi = 1 - \frac{\beta}{\alpha}\theta$ and $\varphi = \frac{\beta}{\alpha}(1-\theta)$ respectively, and $m_2 = m_2\left(B_1, \eta_2, \{\eta_2\}_{\mathcal{E}_2 \in \widetilde{\Psi}}, \frac{\beta}{\alpha}\right)$. The premium is $\rho = -\frac{Cov(m_2, \eta_2)}{\mathbb{E}[m_2]}$.

This proposition establishes that for premia to arise at $t = 1$, we require several ingredients: information must be asymmetric in the FX hedging market, the exchange rate must depend on the private shock at $t = 2$, and the external lenders in the FX spot market must dislike future exchange rate volatility.

We can illustrate the proposition using Figure 1. The figure plots the exchange rate contours as a function of φ on the vertical axis and θ on the horizontal axis, for $\alpha > \beta \geq 0$. Their slope is $-\frac{\beta}{\alpha}$. On panel A of the figure, $\beta = 0$ so the contours are horizontal and the exchange rate increases as we travel upwards. On panel B, $\beta > 0$ so the contours are downward-sloping and the exchange rate increases as we travel upwards and rightwards.

Figure 1: Exchange Rate Contours and Insurability



Note: The blue and red lines represent contours. The shaded region lies between the upper and lower contours $\varphi = 1 - \frac{\beta}{\alpha}\theta$ and $\varphi = \frac{\beta}{\alpha}(1-\theta)$. The dotted arrow shows the direction in which the exchange rate is increasing.

Owing to the tractability of the linear exchange rate function, we can partition the panels using the upper and lower contours given by $\varphi = 1 - \frac{\beta}{\alpha}\theta$ and $\varphi = \frac{\beta}{\alpha}(1-\theta)$ respectively, and reflect that partition in the expression for the lenders' bond position shown in the proposition. The first term in that expression represents the shaded region between these upper and lower contours; within that region, we have plotted solid blue contours. That region covers the whole of panel A but is a parallelogram in panel B. The second and third terms represent the lower and upper triangular unshaded regions in the tails of the exchange rate distribution, and they are present only in panel B; within those regions, we have plotted dashed red contours.

How those regions give rise to insurability and premia depends on which case and subcase identified in the proposition is applicable.

Subcases (a) (i) and (ii) cover the symmetric information environment. Here, insurers and speculators agree on the probabilities of all exchange rates $\mathcal{E}_2$, whether panel A or B is relevant. From proposition 1, it follows that there is full insurability, i.e., $\widetilde{\Psi} = \Psi$. From the lenders' FOC related to insurance, we obtain that $m_2 = 1$ for all $\mathcal{E}_2$ in all the regions of the panels. As a result, the premium ρ is zero irrespective of exchange rate volatility given by the parameters $\{\alpha, \beta\}$. In subcase (a) (ii), θ is known to insurers and speculators at $t = 1$, so a smaller set of exchange rates need to be insured.

By contrast, if there is asymmetric information, the outcome depends on whether panel A or B is relevant.

Subcase (a) (iii) applies if $\beta = 0$ as on panel A. The private information of speculators does not give them superior information about the exchange rate. Accordingly, insurers and speculators continue to agree on the probabilities of all exchange rates, and there is full insurability. We again obtain $m_2 = 1$ in the shaded region, and there are no other regions to consider.

Case (b) applies if $\beta > 0$ as on panel B, and there is partial insurability because the tails of the exchange rate distribution are uninsurable. Insurers and speculators continue to agree on the probabilities of the exchange rates in the shaded region (i.e., this region is the insurable set $\widetilde{\Psi}$, with $m_2 = 1$ within that set coming from the lenders' FOC), but *not* in the two unshaded regions. All the exchange rates in the shaded region can occur for some value of θ, and each value of θ is equally likely. Correspondingly, while the private information of speculators is material (i.e., given θ, they can understand which values of φ need to occur for any particular exchange rate to be achieved), it does not affect their assessed probability of any exchange rate within the region. As a result, while insurers are aware that speculators are more informed, they are still willing to participate in the market. However, the exchange rates in the two unshaded regions are not insurable because insurers know that such exchange rates require the value of θ to be at one extreme and cannot occur for values of θ at the other extreme. Therefore, speculators who know that θ is at one extreme or another can bet against the insurers and make profits at their expense. As a result, insurers are not willing to participate in the market.

Partial insurability may give rise to premia. The premium ρ depends on the covariance of m_2 with the dollar return η_2. Since $m_2 = 1$ within the insurable set, any nonzero covariance must be generated from the uninsurable tails of the exchange rate distribution. The value of m_2 in those regions originates from the lenders' foreign owners and is not pinned down by the lenders' FOC. On the one hand, if $m_2 = 1$ also happens to hold in those tails, the covariance and premium are both zero. On the other hand, if external lenders generally have a higher m_2 during combinations of shocks where the exchange rate is more depreciated and the dollar return η_2 is lower, the covariance is negative and the premium is positive.

The function $m_2\left(B_1, \eta_2, \{\eta_2\}_{\mathcal{E}_2 \in \widetilde{\Psi}}, \frac{\beta}{\alpha}\right)$ outside the insurable set $\widetilde{\Psi}$ is derived from the definition $m_2 \equiv \frac{M_2}{\overline{M}}$. Outside the insurable set, we set $Q_L(\mathcal{E}_2) = 0$ so $M_2(B_1, \eta_2, Q_L(\mathcal{E}_2))$ depends on the reduced set of variables $\{B_1, \eta_2\}$. $\overline{M}$ depends on the variables $\{B_1, \eta_2\}$ within the insurable set $\widetilde{\Psi}$ and the boundaries of that set.

This proposition can be used alongside the definition of the competitive equilibrium above to characterize outcomes as a function of any given monetary policy and information structures.

3 Constrained Efficient Policies

In this section, we show how concerns related to insurability and premia alter the constrained efficient monetary policy and consumption levels.

3.1 Planner Problem

We assume that the planner's information structure is identical to that of the insurers, $\mathcal{I}_I$. The constrained efficient allocation is defined as follows.

Definition 2 (Constrained efficient allocation). *A constrained efficient allocation comprises the quantities* $\left\{\{C_{Ht}, C_{Ft}, N_t, Y_{Ht}\}_{t=1}^{2}, D_1, \{Q_L(\mathcal{E}_2), Q_I(\mathcal{E}_2), Q_S(\mathcal{E}_2)\}_{\mathcal{E}_2\in\Psi}\right\}$*, prices* $\left\{P_H, \{W_t\}_{t=1}^{2},\right.$ $\left.\{p(\mathcal{E}_2)\}_{\mathcal{E}_2\in\Psi}\right\}$*, exchange rate sets* $\left\{\Psi, \widetilde{\Psi}\right\}$*, and probability distributions* $\{\pi(\mathcal{E}_2|\mathcal{I}_I), \pi(\mathcal{E}_2|\mathcal{I}_S)\}_{\mathcal{E}_2\in\Psi}$ *that maximize household welfare under full commitment given the foreigners' SDF function* M_2 *and the information structures* $\{\mathcal{I}_L, \mathcal{I}_I, \mathcal{I}_S\}$*, subject to the restriction that the allocation can be implemented via policy instruments* $\left\{i_1, \{\mathcal{E}_t\}_{t=1}^{2}, \phi, \mu_1\right\}$ *in a competitive equilibrium, where* $\mathcal{E}_2 = \alpha\varphi + \beta\theta + \gamma$ *for some* $\{\alpha, \beta, \gamma\}$.

Following Farhi and Werning (2016) and Basu et al. (2025), we define two wedges which summarize the distance of an allocation from the frictionless first-best frontier:

$$\tau_{Ht} \equiv 1 + \frac{1}{A_t}\frac{U_{Nt}}{U_{Ht}} = 1 - \frac{1}{A_t}\frac{C_{Ht}}{\sigma_H} \text{ for } t \in \{1,2\}$$

$$\tau_{\Gamma 2} \equiv \mathbb{E}\left[\left[\eta_2 - (1+i^*)\right]\frac{\sigma_F}{C_{F2}}\right].$$

The aggregate demand (AD) wedges $\{\tau_{Ht}\}_{t=1}^{2}$ compare the consumption demand for home goods against the labor disutility from the production of those goods. The UIP wedge $\tau_{\Gamma 2}$ measures the expected marginal utility of the consumption loss owing to the premia paid by domestic agents to external lenders.

The planner's period-t utility function V can be written in the following form to incorporate the households' intratemporal FOCs:

$$V\left(C_{Ft}, \frac{\mathcal{E}_t}{P_H}\right) \equiv U\left(\frac{\sigma_H}{\sigma_F}\frac{\mathcal{E}_t}{P_H}C_{Ft}, C_{Ft}, \frac{1}{A_t}\frac{\sigma_H}{\sigma_F}\frac{\mathcal{E}_t}{P_H}C_{Ft}\right).$$

It produces the following partial derivatives:

$$\frac{\partial V}{\partial C_{Ft}} = \frac{\sigma_F}{C_{Ft}}\left[1 + \frac{\sigma_H}{\sigma_F}\tau_{Ht}\right] \text{ and } \frac{\partial V}{\partial\left(\frac{\mathcal{E}_t}{P_H}\right)} = \sigma_H\frac{P_H}{\mathcal{E}_t}\tau_{Ht}.$$

We also define the auxiliary external repayment variable $\chi \equiv (1-\mu_1)(1+i_1)\mathcal{E}_1$ $(= \eta_2\mathcal{E}_2)$, which combines several policy settings from $t=1$ and does not depend on the shocks (φ, θ).

Given these definitions, we can write the planner problem as follows:

$$V^P = \max_{\{C_{F1}, C_{F2}, \mathcal{E}_1, \alpha, \beta, \gamma, \chi, P_H\}} V\left(C_{F1}, \frac{\mathcal{E}_1}{P_H}\right) + \delta \int_{\theta=0}^{1} \int_{\varphi=0}^{1} V\left(C_{F2}, \frac{\mathcal{E}_2}{P_H}\right) \pi\left(\theta | \mathcal{I}_I\right) d\varphi d\theta \tag{1}$$

subject to the constraints:

$$B_0 (1+i^*) = (Y_{F1} - C_{F1}) + (Y_{F2} - C_{F2}) \frac{\mathcal{E}_2}{\chi} \text{ for all } (\varphi, \theta) \tag{2}$$

$$\begin{aligned} 0 = & \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] \pi\left(\theta | \mathcal{I}_I\right) d\varphi d\theta \\ & + \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} m_2 \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] \pi\left(\theta | \mathcal{I}_I\right) d\varphi d\theta \\ & + \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} m_2 \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] \pi\left(\theta | \mathcal{I}_I\right) d\varphi d\theta \end{aligned} \tag{3}$$

$$\mathcal{E}_2 = \alpha\varphi + \beta\theta + \gamma. \tag{4}$$

The planner problem reflects that both the public and private shocks may strike the economy, and it reveals the existence of three externalities we will describe below. Out of the choice variables in the planner's maximand above, the level of imports C_{F2} is set as a function of the state of nature (φ, θ), while the parameters $\{\alpha, \beta, \}$ determine the sensitivity of the exchange rate $\mathcal{E}_2$ to (φ, θ).

The utility function (1) incorporates the productivity level A_2, which may vary with (φ, θ) at $t = 2$. There is an *AD externality*:[14] households do not internalize the impact of their individual consumption decisions on the AD wedges $\{\tau_{Ht}\}_{t=0}^{2}$.

There is a continuum of resource constraints (2), one for each state of nature (φ, θ), obtained by combining the households' budget constraints with the market clearing conditions and the expression for the lump sum rebates. For $t = 1$, we fix the dollar value of repayments on inherited external debt to $B_0(1+i^*)$. At $t = 2$, the endowment of tradable goods Y_{F2} may depend on (φ, θ). Since all the external debt B_1 is in local currency, any variation in $\mathcal{E}_2$ across states of nature alters the dollar value of repayments on that debt across states, which in turn affects C_{F2}.

The premium constraint (3) aggregates the repayments on external debt across states of nature at $t = 2$ to ensure that external lenders receive the necessary premium to hold that external debt at

[14] As described in Farhi and Werning (2016), such an externality naturally emerges in the New Keynesian literature when considering desirable monetary policies.

$t = 1$. The constraint comes from the FX spot and hedging markets as summarized in Proposition 2. If case (a) of Proposition 2 applies, there is no premium. If case (b) applies, there may be a *local currency (LC) premium externality*:[15] households do not internalize that their individual borrowing decisions may affect economywide premia and thereby the UIP wedge $\tau_{\Gamma 2}$. The mechanism is that their borrowing affects B_1, which in turn affects m_2 in the uninsurable regions of exchange rates.

The exchange rate function (4) relates the exchange rate $\mathcal{E}_2$ to the public and private shocks. Combined with the premium constraint (3), it reveals that in our environment, there may be a novel *information sensitivity (IS) externality*: when agents respond to the shocks (φ, θ) at $t = 2$, they do not internalize that their individual actions may affect monetary policy and exchange rates in a manner that generates premia. Moreover, even an optimizing planner may not be able to avoid these premia, as they take the agents' optimization conditions as given. If the planner sets β above zero, such that $\mathcal{E}_2$ responds to the private shock at $t = 2$, and if the information structures feature $\mathcal{I}_I = \mathcal{I}_\emptyset$ and $\mathcal{I}_S = \mathcal{I}_\theta$, there may be a switch from case (a) to case (b) of Proposition 2. Within case (b), any decision by the planner to alter the parameters $\{\alpha, \beta\}$ affects the boundaries of the insurable set and the resulting premium. The premium also varies with any changes in the parameters $\{\alpha, \beta, \gamma, \chi\}$ affecting m_2 and/or $\frac{\chi}{\mathcal{E}_2}$ in the uninsurable regions.

The households' Euler condition and the firms' price-setting equation do not appear in the planner problem above, but remain operational in the background. Once the planner selects the variables indicated above, the households' Euler condition enables us to back out the implicit policy rate i_1 and capital inflow tax μ_1, while the price-setting equation determines the implicit labor tax ϕ.

3.2 First Order Conditions

We characterize the planner problem via the first order approach and assume that the solution satisfies $\alpha > \beta \geq 0$.[16] In this subsection, we rearrange the planner's FOCs to derive four insights for the constrained efficient allocation. We use these insights to construct some general results in subsection 3.3 and explore them further via specific applications in section 4.

[15] Such an externality naturally emerges when considering desirable policies in papers focusing on the portfolio balance approach to FX markets (e.g., Cavallino, 2019; Fanelli and Straub 2021; and Itskhoki and Mukhin, 2023).

[16] The first-order approach assumes that the problem has a unique interior solution for all variables and that the planner's FOCs establish this welfare maximum. Appendix B contains the full set of FOCs. The FOC for P_H turns out to be redundant, so we normalize $P_H = 1$ throughout the remainder of the paper. Note that for subcase (a) (ii), the inequality $\alpha > \beta \geq 0$ is both trivially satisfiable (since the value of $\beta\theta + \gamma$ is determined but the precise value of β is not) and irrelevant (as there is full insurability). The appendix shows how the FOCs are amended when $\beta > \alpha \geq 0$ and also when there are no premia and the exchange rate function is unrestricted.

The first insight is that the AD wedge is zero at $t = 1$:

$$\tau_{H1} = 0. \tag{5}$$

This result comes directly from the FOC for $\mathcal{E}_1$. It means that irrespective of the level of import consumption C_{F1}, the planner uses exchange rate flexibility to set C_{H1} to the value that eliminates the AD externality and its associated wedge in that period.

The second insight is that the level of the external repayment variable χ is set in a manner that internalizes AD and IS externalities:

$$B_1 \int_{\theta=0}^{1} \int_{\varphi=0}^{1} \frac{1}{\mathcal{E}_2} \frac{\delta \sigma_F}{C_{F2}} \left[1 + \frac{\sigma_H}{\sigma_F} \tau_{H2}\right] \pi(\theta|\mathcal{I}_I)\, d\varphi d\theta = \Lambda \left[\int_{\theta=0}^{1} \int_{\varphi=0}^{1} \frac{1}{\mathcal{E}_2} \pi(\theta|\mathcal{I}_I)\, d\varphi d\theta + z_\chi\right] \tag{6}$$

$$\text{where } B_1 = B_0 (1 + i^*) + (C_{F1} - Y_{F1}) = (Y_{F2} - C_{F2}) \frac{\mathcal{E}_2}{\chi}$$

$$z_\chi = \begin{cases} \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} \left\{[m_2 - 1] \frac{1}{\mathcal{E}_2} + \frac{\partial m_2}{\partial \chi} \left[\frac{\chi}{\mathcal{E}_2} - (1 + i^*)\right]\right\} \pi(\theta|\mathcal{I}_I)\, d\varphi d\theta \\ + \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} \left\{[m_2 - 1] \frac{1}{\mathcal{E}_2} + \frac{\partial m_2}{\partial \chi} \left[\frac{\chi}{\mathcal{E}_2} - (1 + i^*)\right]\right\} \pi(\theta|\mathcal{I}_I)\, d\varphi d\theta, \end{cases}$$

and Λ is the multiplier on the premium constraint (3). This equation comes from combining the FOCs for χ and C_{F2}. The equation reveals that Λ is non-zero only if the external debt B_1 is also non-zero. If $B_1 = 0$, changes in the external repayment variable χ do not affect C_{F2} via the resource constraints (2), so χ is set solely to make the premium constraint (3) non-binding, i.e., $\Lambda = 0$. Instead, if $B_1 \neq 0$, changes in χ affect C_{F2} and thereby welfare.

If we remove the externalities so that $\tau_{H2} = z_\chi = 0$, equation (6) is simplified. In this case, the equation shows that χ is set to balance two considerations: the left hand side reflects that an increase in χ causes a decrease in C_{F2} if the economy has positive external debt, i.e., $B_1 > 0$, while the right hand side reflects that such an increase may be needed to generate sufficient returns for external lenders to hold the debt at $t = 1$.

However, externalities generally exist and yield additional considerations. AD externalities at $t = 2$ are captured by the wedges τ_{H2} on the left hand side of equation (6), reflecting that changes in C_{F2} can induce inefficient changes in these wedges. The IS externality is captured by the term z_χ on the right hand side of the equation. This term incorporates that an increase in χ has additional effects on the SDF-weighted returns of external lenders because of the uninsurable regions of the exchange rate distribution. In the definition of the variable z_χ, the two rows capture the two uninsurable regions. Within each row, the first term reflects that there is an increase in external repayments in regions where m_2 may differ from 1, while the second term reflects that the values of m_2 within these regions may themselves change.

The third insight is that the levels of import consumption C_{Ft} are set in a manner that internalizes AD and LC premium externalities:

$$\frac{\sigma_F}{C_{F1}} = \int_{\theta=0}^{1}\int_{\varphi=0}^{1} \frac{\chi}{\mathcal{E}_2}\frac{\delta\sigma_F}{C_{F2}}\left[1+\frac{\sigma_H}{\sigma_F}\tau_{H2}\right]\pi\left(\theta|\mathcal{I}_I\right)d\varphi d\theta + \Lambda z_{F1} \tag{7}$$

$$\text{where } z_{F1} = -\left[\begin{array}{c} \int_{\theta=0}^{1}\int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} \frac{\partial m_2}{\partial B_1}\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]\pi\left(\theta|\mathcal{I}_I\right)d\varphi d\theta \\ +\int_{\theta=0}^{1}\int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} \frac{\partial m_2}{\partial B_1}\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]\pi\left(\theta|\mathcal{I}_I\right)d\varphi d\theta \end{array}\right].$$

This equation comes from combining the FOCs for C_{F1} and C_{F2}.

If we remove the externalities so that $\tau_{H2} = z_{F1} = 0$, equation (7) is simplified. In this case, the equation shows that the profile of imports C_{Ft} over time is determined solely by the expected dollar value of repayments on external debt, i.e., $\frac{\chi}{\mathcal{E}_2}$.

Externalities add terms to the right hand side of the equation. AD externalities at $t = 2$ are captured by the wedges τ_{H2}. The LC premium externality is captured by the term z_{F1}. This term incorporates that an increase in C_{F1} causes an increase in B_1 and thereby changes in m_2 in the uninsurable regions of the exchange rate distribution.

The fourth insight is that the parameters $\{\alpha, \beta, \gamma\}$ in the exchange rate function are set in a manner that internalizes AD and IS externalities. For each parameter $\omega \in \{\alpha, \beta, \gamma\}$, we obtain:

$$\begin{aligned} &\int_{\theta=0}^{1}\int_{\varphi=0}^{1} \frac{\partial\mathcal{E}_2}{\partial\omega}\left\{\frac{1}{\mathcal{E}_2}\delta\sigma_H\tau_{H2} + B_1\frac{\chi}{\left(\mathcal{E}_2\right)^2}\frac{\delta\sigma_F}{C_{F2}}\left[1+\frac{\sigma_H}{\sigma_F}\tau_{H2}\right]\right\}\pi\left(\theta|\mathcal{I}_I\right)d\varphi d\theta \\ &= \Lambda\left[\int_{\theta=0}^{1}\int_{\varphi=0}^{1}\frac{\partial\mathcal{E}_2}{\partial\omega}\frac{\chi}{\left(\mathcal{E}_2\right)^2}\pi\left(\theta|\mathcal{I}_I\right)d\varphi d\theta + z_{\omega1} + z_{\omega2}\right] \end{aligned} \tag{8}$$

$$\text{where } z_{\omega1} = \left\{\begin{array}{l} \int_{\theta=0}^{1}\int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)}\left\{\left[m_2-1\right]\frac{\partial\mathcal{E}_2}{\partial\omega}\frac{\chi}{\left(\mathcal{E}_2\right)^2} - \frac{\partial m_2}{\partial\omega}\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]\right\}\pi\left(\theta|\mathcal{I}_I\right)d\varphi d\theta \\ +\int_{\theta=0}^{1}\int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1}\left\{\left[m_2-1\right]\frac{\partial\mathcal{E}_2}{\partial\omega}\frac{\chi}{\left(\mathcal{E}_2\right)^2} - \frac{\partial m_2}{\partial\omega}\left[\frac{\chi}{\mathcal{E}_2}-(1+i^*)\right]\right\}\pi\left(\theta|\mathcal{I}_I\right)d\varphi d\theta \end{array}\right.$$

$$z_{\omega2} = \frac{\partial\left(-\frac{\beta}{\alpha}\right)}{\partial\omega}\left\{\begin{array}{c} \left[\lim\limits_{\varphi\to\left(\frac{\beta}{\alpha}(1-\theta)\right)^-} m_2 - 1\right]\left[\frac{\chi}{\beta+\gamma}-(1+i^*)\right]\int_{\theta=0}^{1}(1-\theta)\,\pi\left(\theta|\mathcal{I}_I\right)d\theta \\ +\left[\lim\limits_{\varphi\to\left(1-\frac{\beta}{\alpha}\theta\right)^+} m_2 - 1\right]\left[\frac{\chi}{\alpha+\gamma}-(1+i^*)\right]\int_{\theta=0}^{1}\theta\pi\left(\theta|\mathcal{I}_I\right)d\theta \end{array}\right\}.$$

This equation comes from combining the FOCs for each parameter and C_{F2}.

If we remove the externalities so that $\tau_{H2} = z_{\omega 1} = z_{\omega 2} = 0$, equation (8) is simplified and has a similar interpretation to that of equation (6). Any change in the parameters $\{\alpha, \beta, \gamma\}$ causes movements in the exchange rates $\mathcal{E}_2$ across all states of nature (φ, θ) at $t = 2$. The left hand side of equation (8) contains just one term, and this term reflects that these movements in $\mathcal{E}_2$ affect C_{F2} via the resource constraints (2) if $B_1 \neq 0$. The right hand side of the equation reflects that the movements in $\mathcal{E}_2$ affect whether there are sufficient returns for external lenders to hold the debt at $t = 1$.

Again, externalities generally exist and yield additional considerations.

AD externalities at $t = 2$ are captured by the wedges τ_{H2} in the two terms on the left hand side of equation (8). The first term reflects that irrespective of the level of B_1, $\mathcal{E}_2$ directly affects τ_{H2} through the households' intratemporal FOCs, while the second term reflects that if $B_1 \neq 0$, there is an additional effect of $\mathcal{E}_2$ on C_{F2}, and thereby on τ_{H2}, via the resource constraints (2).

The first term capturing the IS externality is the term $z_{\omega 1}$ on the right hand side of equation (8). This term incorporates that if $\beta > 0$ to begin with, any change in the parameters $\{\alpha, \beta, \gamma\}$ has additional effects on the SDF-weighted returns of external lenders because of the uninsurable regions of the exchange rate distribution. In the definition of $z_{\omega 1}$, the first term in each row reflects that there are movements in exchange rates $\mathcal{E}_2$ in regions where m_2 may differ from 1, while the second term reflects that the values of m_2 within these regions may themselves change.

The second term capturing the IS externality is the term $z_{\omega 2}$ on the right hand side of the equation. It turns out that this term is non-zero only for the parameters $\{\alpha, \beta\}$, because these parameters determine the boundaries of the insurable set of exchange rates $\widetilde{\Psi}$. In the definition of $z_{\omega 2}$, the term in front of the integral captures the effect of varying the parameters on the slope and size of the insurable set. On Figure 1, the slope of the upper and lower contours are $-\frac{\beta}{\alpha}$. Correspondingly, an increase in β makes the slopes steeper and shrinks the insurable set. Moreover, if $\beta > 0$ to begin with, a decrease in α has a similar effect on the insurable set. The result is an expansion of the two uninsurable regions. The terms inside the integral capture that in the premium constraint (3), the expansion of the uninsurable regions means that more of the returns to external lenders are multiplied by their uninsured SDFs. Consequently, the premium ρ needed to satisfy the constraint may change.

3.3 Policy Impact of Uninsurability

The externalities in equations (6)-(8), and therefore the constrained efficient policy mix, depend on which case and subcase from Proposition 2 applies.

Lemma 1 (Effective information symmetry). *For a country with full insurability $\widetilde{\Psi} = \Psi$, the constrained efficient allocation features no LC and IS externalities, with one exception. Specifically, $z_\chi = z_{F1} = z_{\omega 1} = z_{\omega 2} = 0$ for all $\omega \in \{\alpha, \beta, \gamma\}$ in all subcases of case (a), except that $z_{\beta 2}$ may be non-zero for subcase (a) (iii).*

This result establishes that the LC premium and IS externalities are generally zero throughout case (a) because of full insurability. Subcases (a) (i) and (ii) feature full insurability arising from the symmetric information environment, irrespective of the value of β. Subcase (a) (iii) occurs when there is asymmetric information but the planner happens to choose $\beta = 0$. It features full insurability because the private information of speculators does not give them superior information about the exchange rate. Of course, in this subcase, moving β above zero may generate an IS externality.

The next lemma identifies whether the constrained efficient allocations selected for the subcases (a) (i) and (ii) with symmetric information remain feasible after an episode which triggers asymmetric information at $t = 1$. The resource constraints (2) remain satisfied. Accordingly, whether the constrained efficient allocations remain feasible depends on whether the premium constraint (3) continues to be satisfied.

Lemma 2 (Information asymmetry episode). *Consider a change from $\mathcal{I}_I = \mathcal{I}_S \in \{\mathcal{I}_\emptyset, \mathcal{I}_\theta\}$ to $\mathcal{I}_I = \mathcal{I}_\emptyset$ and $\mathcal{I}_S = \mathcal{I}_\theta$. The premium ρ from the constrained efficient allocation prior to the change in the information structures is too low to satisfy constraint (3) after the change in the information structures if and only if the following condition is satisfied:*

$$\begin{aligned}
0 > & \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[1 - \pi\left(\theta|\mathcal{I}_I\right)\right] \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta \\
& + \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} \left[m_2^{(b)} - \pi\left(\theta|\mathcal{I}_I\right)\right] \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta \\
& + \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} \left[m_2^{(b)} - \pi\left(\theta|\mathcal{I}_I\right)\right] \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta, \qquad (9)
\end{aligned}$$

where the $m_2^{(b)}$ function corresponds to case (b) after the change in information structures, but all the values of the planner's choice variables correspond to the allocation prior to the change.

This lemma allows us to identify three possible scenarios arising from this episode.

The first scenario is that the country begins in subcase (a) (i) and the planner selects $\beta = 0$ in the constrained efficient allocation for that subcase. The change from symmetric to asymmetric information involves speculators becoming more informed than they were before, while external lenders and insurers remain uninformed. The change moves the country to subcase (a) (iii). It does not violate the premium constraint (3) because the entire exchange rate distribution remains insurable at the previous variable values, i.e., $\widetilde{\Psi} = \Psi$ and $m_2 = 1$ in all states of nature (φ, θ).

The second scenario is that the country begins in subcase (a) (i) and the planner selects $\beta > 0$. The change in the information structure is now important for the constrained efficient allocation because it results in uninsurable regions of the exchange rate distribution, i.e., $\widetilde{\Psi} \subset \Psi$. The country moves to case (b). Whether the premium ρ from subcase (a) (i) is sufficient to satisfy the premium constraint (3) in case (b) depends on the values of $m_2^{(b)}$ in the uninsurable regions, as with partial insurability, these values are no longer necessarily equal to one.

Condition (9) is the criterion for the premium ρ to no longer be sufficient. For this second scenario, we have $\pi\left(\theta|\mathcal{I}_I\right) = 1$ for all θ prior to the change in the information structure, so the first term on the right hand side of the condition is zero and only the second and third terms may be non-zero. They correspond to the uninsurable regions. The second term reflects that in the absence of insurance, external lenders gain from the higher dollar value of returns in the lower uninsurable region, as the values of $\eta_2 \equiv \frac{\chi}{\mathcal{E}_2}$ are highest in that region and $\eta_2 > (1 + i^*)$ prevails in some of the region. The third term reflects that the lenders lose from the lower dollar value of returns in the upper uninsurable region, as the values of η_2 are lowest in that region and $\eta_2 < (1 + i^*)$ prevails in some of the region. The inequality in the condition identifies when the external lenders gain less than they lose, e.g., if $m_2^{(b)}$ tends to be smaller in the lower insurable region and larger in the upper insurable region.

The third scenario is that the country begins in subcase (a) (ii). The change from symmetric to asymmetric information involves speculators remaining informed, while external lenders and insurers become less informed than they were before. Condition (9) is again the criterion for the previous premium ρ to no longer be sufficient. For this third scenario, we have $\pi\left(\theta|\mathcal{I}_I\right) = 1$ only for the known value of θ and $= 0$ for other values, so all three terms on the right hand side of the condition may now be non-zero. This amendment reflects that in subcase (a) (ii), external lenders must receive a premium which is appropriate given the specific known value of θ, while in case (b), they must receive a premium which is appropriate given a larger set of possible values of θ.

Next, we turn to how asymmetric information alters the setting of exchange rates at the constrained efficient allocation. The following proposition summarizes the insights above and establishes a generic result.

Proposition 3 (Modified monetary policy). *The constrained efficient allocation for the planner problem maximizing the utility function (1) subject to the constraints (2)-(4) must satisfy the FOCs (5)-(8). Suppose that for $\mathcal{I}_I = \mathcal{I}_S = \mathcal{I}_\emptyset$, the planner selects $\frac{\beta}{\alpha} \in (0,1)$ and $B_1 > 0$. For $\mathcal{I}_I = \mathcal{I}_\emptyset$ and $\mathcal{I}_S = \mathcal{I}_\theta$, there exist $m_2^{(b)}$ functions such that the planner selects a different value of $\frac{\beta}{\alpha}$.*

Our modeling of the FX hedging market results in a planner problem with a tractable modification to the standard set of constraints and a single ratio $\frac{\beta}{\alpha}$ summarizing the insurable set $\widetilde{\Psi}$. Accordingly, it permits the constrained efficient allocation to be characterized using the standard method of building a system of equations which combines the constraints (2)-(4) with interpretable FOCs (5)-(8).

If the country is in case (b), the $m_2^{(b)}$ function affects this system of equations. For the function $m_2^{(b)} = 1$, the planner selects the same value of $\frac{\beta}{\alpha}$ in the asymmetric information case (b) as they do in the symmetric information subcase (a) (i). The condition $\frac{\beta}{\alpha} \in (0,1)$ reflects the assumption that $\alpha > \beta > 0$, while the condition $B_1 > 0$ ensures that the Λ multiplier on the premium constraint is positive. Perturbations in the $m_2^{(b)}$ function alter the system of equations via specific channels that correspond to where the function appears in the system, i.e., via the premium constraint (3) and the LC and IS externalities $\{z_\chi, z_{F1}, z_{\omega 1}, z_{\omega 2}\}$ in the FOCs (6)-(8), where $\omega \in \{\alpha, \beta, \gamma\}$. As a result, they cause changes in all the planner's choice variables and the ratio $\frac{\beta}{\alpha}$ (except in knife-edge cases), with the specific direction and magnitude of their changes being determined by a weighted combination of all these channels.

A change in the ratio $\frac{\beta}{\alpha}$ means a change in the relative sensitivity of the exchange rate $\mathcal{E}_2$ to the private shock θ vis-à-vis the public shock φ. The takeaway is that in our framework, the planner generally changes how they stabilize the economy with respect to the shocks (φ, θ) once they have concerns about the insurability of exchange rates and the associated UIP premium.

The next result states that when faced with insurability concerns, the planner may reduce the relative sensitivity of the exchange rate to the private shock.

Corollary 1 (Aloof central banker). *Suppose that for $\mathcal{I}_I = \mathcal{I}_S = \mathcal{I}_\emptyset$, the planner selects $\frac{\beta}{\alpha} \in (0,1)$ and $B_1 > 0$. For $\mathcal{I}_I = \mathcal{I}_\emptyset$ and $\mathcal{I}_S = \mathcal{I}_\theta$, there exist $m_2^{(b)}$ functions such that the planner selects a lower value of $\frac{\beta}{\alpha}$. One way to implement the constrained efficient allocation is to delegate monetary policy to a central banker who can commit to this lower value of $\frac{\beta}{\alpha}$.*

The first part of the corollary states that for some $m_2^{(b)}$ functions, the planner selects a lower value of the ratio $\frac{\beta}{\alpha}$ in the asymmetric information case (b) than they do in the symmetric information subcase (a) (i). The insurable set's boundaries in case (b) have slopes $-\frac{\beta}{\alpha}$, so a decrease in $\frac{\beta}{\alpha}$ makes the slopes gentler and expands the insurable set. We can provide a partial yet instructive insight into this result by considering the effect of a small perturbation in the $m_2^{(b)}$ function on the IS externality term $z_{\beta 2}$, which is present in the FOC for β. Consider the following perturbation in case (b):

$$m_2^{(b)}\left(B_1, \eta_2, \{\eta_2\}_{\mathcal{E}_2 \in \widetilde{\Psi}}, \frac{\beta}{\alpha}\right) = 1 + \epsilon \widetilde{m}_2\left(B_1, \eta_2, \{\eta_2\}_{\mathcal{E}_2 \in \widetilde{\Psi}}, \frac{\beta}{\alpha}\right).$$

At $\epsilon = 0$, the marginal impact of this perturbation on the term $z_{\beta 2}$ is as follows:

$$\frac{dz_{\beta 2}}{d\epsilon} = -\frac{1}{2\alpha}\left\{\begin{array}{c} \left[\frac{\chi}{\beta+\gamma} - (1+i^*)\right] \lim\limits_{\varphi \to \left(\frac{\beta}{\alpha}(1-\theta)\right)^-} \widetilde{m}_2 \\ + \left[\frac{\chi}{\alpha+\gamma} - (1+i^*)\right] \lim\limits_{\varphi \to \left(1-\frac{\beta}{\alpha}\theta\right)^+} \widetilde{m}_2 \end{array}\right\}.$$

The term $z_{\beta 2}$ captures how much an *increase* in β causes a decrease in the SDF-weighted dollar returns to external lenders by expanding the size of the uninsurable regions in Figure 1, which in turn makes the premium constraint (3) more difficult to satisfy. We can see that the perturbation of $m_2^{(b)}$ makes the term $z_{\beta 2}$ larger if the term inside the curly brackets is negative. There exist $\widetilde{m}_2$ functions such that the integral is indeed negative, for example when $\widetilde{m}_2 < 0$ and $\frac{\chi}{\beta+\gamma} > (1+i^*)$ at the boundary of the lower insurable region, and/or $\widetilde{m}_2 > 0$ and $\frac{\chi}{\alpha+\gamma} < (1+i^*)$ at the boundary of the upper insurable region. Such an outcome may occur for perturbations that make foreigners worse off after depreciations. If the perturbation makes the term $z_{\beta 2}$ larger in the FOC for β and nothing else changes, the planner tends to desire to *decrease* β and thereby decrease the ratio $\frac{\beta}{\alpha}$.

To establish the direction of change of $\frac{\beta}{\alpha}$, the above term cannot be viewed in isolation. The change in $\frac{\beta}{\alpha}$ is determined by a weighted combination of the effects of the perturbation as it works through the multiple channels described above for Proposition 3. In subsection 4.1, we simulate an example $m_2^{(b)}$ function where the ratio $\frac{\beta}{\alpha}$ does decrease in case (b) relative to subcase (a) (i).

The reduction in $\frac{\beta}{\alpha}$ evokes and yet contrasts with a separate literature on information sensitivity. Dang et al. (2013, 2015) examine how private agents can design securities to make them less sensitive to private information and thereby more liquidly tradable by uninformed agents. By contrast, our framework takes as given that domestic agents issue local currency debt securities, and then it explores how monetary policy should be modified to affect the sensitivity of the dollar returns on that debt to the private shock: the less sensitive they are, the lower the premium.

Our monetary policy result differs from other papers in the portfolio balance literature because of the different way in which we model FX market premia. In Basu et al. (2025), premia are affected by external debt but not by exchange rate volatility, so their policy mix incorporates the LC premium externality but does not otherwise modify exchange rates. In Itskhoki and Mukhin (2023), premia are generated by exchange rate volatility as none of it is insurable, while in Chernov et al. (2024), financial assets from outside the FX market can be used to hedge risks. By adding the notion of endogenous insurability in the FX hedging market, we show that premia depend specifically on the volatility of exchange rates with respect to shocks over which there is asymmetric information, while other forms of volatility can be insured away. As a result, our constrained efficient monetary policy may include a lower sensitivity of the exchange rate to the private shock vis-à-vis the public shock.

The second part of the corollary reflects the importance of commitment for the constrained efficient allocation. In the planner's FOCs, all the terms $\{z_\chi, z_{F1}, z_{\omega 1}, z_{\omega 2}\}$ are multiplied by Λ, the multiplier on the premium constraint (3) which must be satisfied at $t = 1$. Accordingly, the implementation of the constrained efficient allocation relies on the planner's ability to commit at $t = 1$ to its exchange rate function at $t = 2$.

If the planner cannot commit to their future actions in such a manner, the constrained efficient allocation can still be implemented by delegating monetary policy to a central banker who has the ability to commit to future actions and also follows the same preferences and information structure as the planner. In the presence of insurability concerns, our framework provides a case for selecting an "aloof central banker" who commits to down-weight the private shock θ in their exchange rate function in case (b) relative to subcase (a) (i). This case for monetary delegation is distinct from the inflation bias rationale in Rogoff (1985), as it relies on information sensitivity rather than any fear of excessive inflation.

4 Applications

In this section, we illustrate the adaptability of the general model by exploring four applications of it which can shed light on different kinds of policies.

4.1 Composition of External Lenders

While the general model permits a broad range of possibilities for the modeling of the external lenders, here we explore two options. The first option is that the lenders are owned by foreigners with log utility. The second option is that the lenders are owned by foreigners who face binding financial constraints after large losses but not otherwise.

For our first option, we assume that the lenders are owned by foreigners who maximize the following welfare:

$$\max_{C^*_{F2}} \mathbb{E}\left[\log C^*_{F2} | \mathcal{I}_L\right],$$

subject to the budget constraint:

$$\kappa \Pi_2 + Y^*_{F2} \geq C^*_{F2},$$

where $\kappa > 0$ is the ratio of the measure of the home country relative to the measure of the lenders' foreign owners, $\Pi_2 = [\eta_2 - (1 + i^*)] B_1 + Q_L(\mathcal{E}_2)$ is the dollar profit of the lenders as described in subsection 2.1, Y^*_{F2} is the foreign endowment of tradable goods, and C^*_{F2} is the foreign consumption of tradable goods. The foreigners' SDF M_2 reflects the marginal value of Π_2 derived from this maximization problem.

The following result derives the corresponding function m_2 and its implication for the premium.

Proposition 4 (Log utility). *Suppose that the modeling of the external lenders follows our first option. For a country in case (b) with $\kappa > 0$, the function m_2 is as follows:*

$$m_2 = \begin{cases} 1 & \text{for } \mathcal{E}_2 \in \widetilde{\Psi} \\ \dfrac{\frac{\kappa}{1-\frac{\beta}{\alpha}} B_1 \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta + Y^*_{F2}}{\kappa B_1 \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] + Y^*_{F2}} & \text{for } \mathcal{E}_2 \notin \widetilde{\Psi}, \end{cases} \tag{10}$$

For any given parameters $\{\alpha, \beta, \gamma\}$, the premium constraint (3) yields $\frac{d\chi}{dB_1} > 0$.

This proposition relates the premium in case (b) to each lender's exposure to the country's external debt. This exposure is determined as follows.

First, the values of m_2 in the uninsurable regions deviate from one only if $\kappa > 0$, i.e., if the country is large relative to the lenders' foreign owners. Such a situation may arise for a range of countries. For example, it could arise for a small country whose external debt can only be held by an even smaller set of foreigners who specialize in that country's debt. Alternatively, it could arise for a large country whose external debt must be held by foreigners located in smaller countries. By contrast, if the country is small and the set of foreign owners is large, i.e., $\kappa \to 0$, we obtain $m_2 = 1$ throughout the insurable and uninsurable regions, so there is no premium.

Second, for $\kappa > 0$, the level of the country's external debt B_1 is important for determining the premium. The second part of the proposition establishes that for any given parameters $\{\alpha, \beta, \gamma\}$ within the exchange rate function (4), the planner faces a premium schedule that is upward-sloping in external debt. Specifically, they must offer a higher external repayment χ if they choose to increase the consumption of imports C_{F1} and correspondingly issue more debt B_1 at $t = 1$.

An implication of our model's finding is that if the country switches from full to partial insurability without a change in the parameters $\{\alpha, \beta, \gamma\}$, the jump in the premium is increasing in the level of external debt. If the external debt is low, the jump in the premium would be small, so an episode of partial insurability may not be clearly detectable in practice. However, if the external debt is large, the jump may be large and noticeable.

Figures 2 and 3 show simulations for the comparative statics with respect to κ and B_1 respectively. The results in both figures are consistent with Corollary 1 and Proposition 4. We use the example functions:

$$Y_{F1} = \overline{Y}, Y_{F2} = \overline{Y} - \xi\left(\varphi - \frac{1}{2}\right) - \nu\left(\theta - \frac{1}{2}\right), \text{ and } A_1 = A_2 = \overline{A}. \tag{11}$$

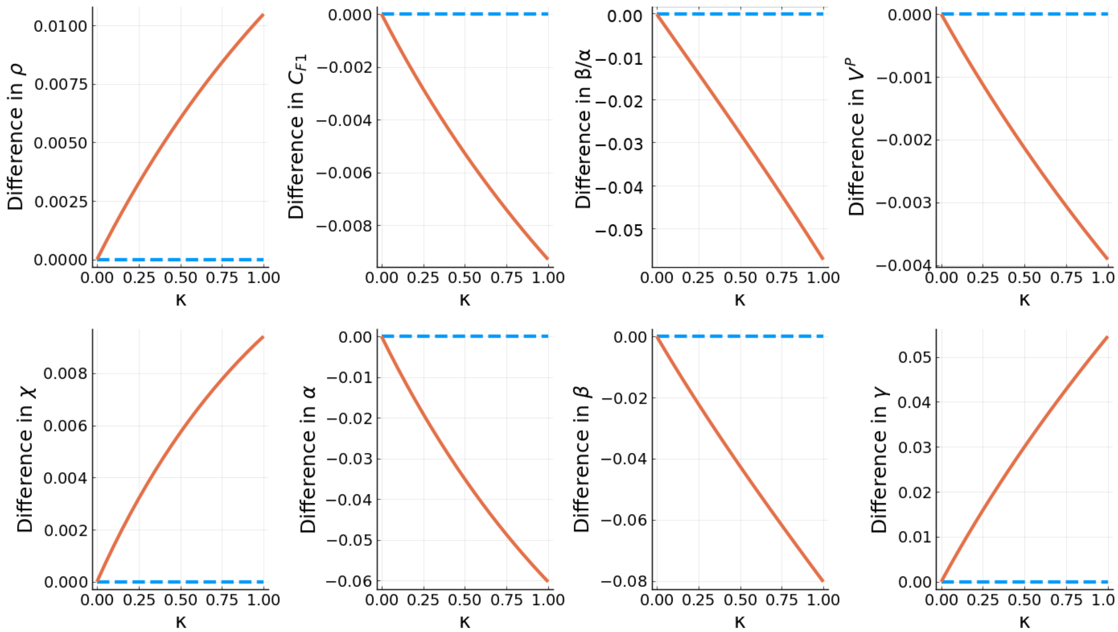
The red lines in the figures plot the difference for the constrained efficient variables going from the full insurability subcase (a) (i) to the partial insurability case (b). Both figures show that insurability problems cause a higher premium ρ, lower imports C_{F1}, a lower value of $\frac{\beta}{\alpha}$, and lower welfare V^P. Both α and β are lower in case (b), but the latter is reduced relatively more than the former so that $\frac{\beta}{\alpha}$ is lower. Figure 2 shows that these problems become more severe as the ratio κ increases, while figure 3 shows that they become more severe as the inherited external debt B_0 increases.

Next, we consider an extension of the model with two categories of external lenders.

Corollary 2 (Inframarginal lenders). *Suppose that the marginal lenders follow our first modeling option but there also exist inframarginal lenders who hold the dollar value S_1 of local currency bonds. Proposition 4 continues to hold but with B_1 replaced by $B_1 - S_1$.*

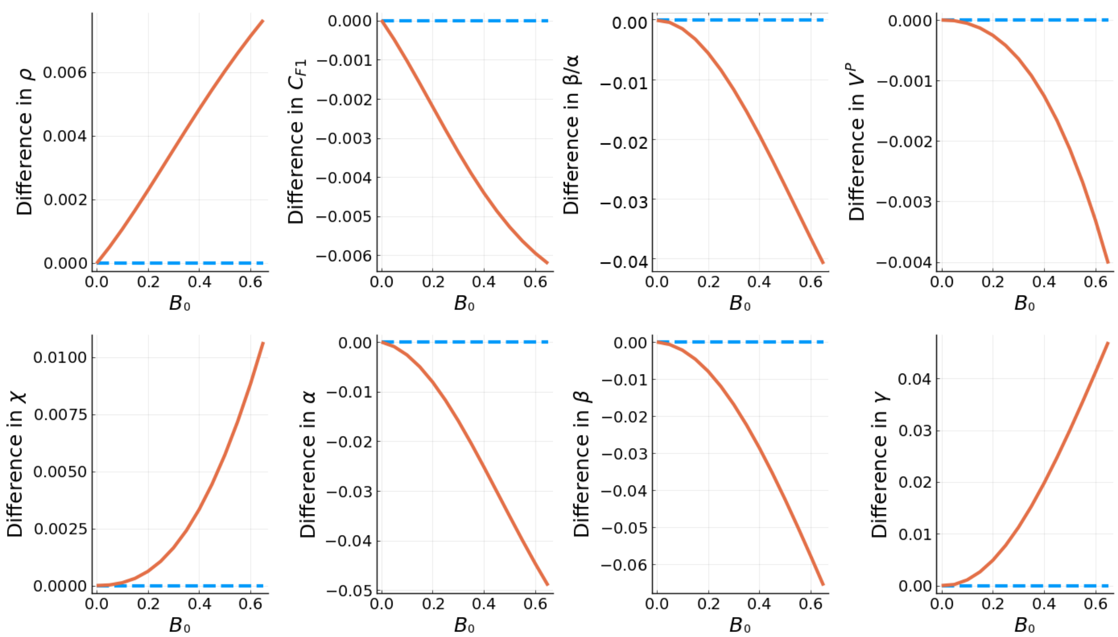
This corollary establishes a link between our papers and others which highlight the role of inframarginal return-insensitive investors in FX markets. Some papers label these investors as "noise traders" and posit that they cause high-frequency variations in FX market premia (e.g., Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021, 2023; and Basu et al., 2025). Here, we do not explore the stochastic properties of such investors. Instead, we simply observe that consistent with those papers, the premium may be higher in our framework if inframarginal investors hold less of the external debt, because each marginal investor must then be more exposed to the debt. Our framework also highlights the importance of insurability: inframarginal investors affect premia only if some exchange rates are uninsurable.

Figure 2: How Insurability Problems Change Allocations, Given the Value of κ



Note: Difference in variable = Constrained efficient value in case (b) minus value in subcase (a) (i). Parameter values are $\sigma_H = 0.5, \delta = 0.96, i^* = \frac{1}{\delta} - 1, B_0 = 0.5, Y^*_{F2} = 1, \xi = 0.6, \nu = 0.3,$ and $\overline{A} = 1$.

Figure 3: How Insurability Problems Change Allocations, Given the Value of B_0



Note: Difference in variable = Constrained efficient value in case (b) minus value in subcase (a) (i). Parameter values are $\sigma_H = 0.5, \delta = 0.96, i^* = \frac{1}{\delta} - 1, \kappa = 0.5, Y^*_{F2} = 1, \xi = 0.6, \nu = 0.3,$ and $\overline{A} = 1$.

For our second modeling option of the external lenders, we assume that they are owned by foreigners who maximize the following welfare:

$$\max_{C^*_{F2}} \mathbb{E}\left[\left.\frac{C^{*1-\nu}_{F2}}{1-\nu}\right| \mathcal{I}_L\right],$$

subject to the budget constraint:

$$\Pi_2 - \varsigma\Theta\left(\Pi_2\right)\frac{\left(\underline{\Pi}-\Pi_2\right)^2}{2} + Y^*_2 \geq C^*_{F2},$$

$$\text{where } \Theta\left(\Pi_2\right) = \begin{cases} 0 & \text{if } \Pi_2 \geq \underline{\Pi} \\ 1 & \text{if } \Pi_2 < \underline{\Pi} \end{cases} \quad \text{for some } \underline{\Pi} < 0.$$

The indicator function $\Theta\left(\Pi_2\right)$ captures in reduced form a one-sided financial constraint, e.g., costly margin calls when losses are high.

The corresponding function m_2 reflects the asymmetry of the financial constraint.

Proposition 5 (One-sided constraint). *Suppose that the modeling of the external lenders follows our second option and that for $\varsigma = 0$, the constrained efficient allocation for case (b) features $\frac{\beta}{\alpha} \in (0,1)$ and $B_1 > 0$. There exist values of $\underline{\Pi} < 0$ such that for $\varsigma > 0$, the constrained efficient allocation features $\frac{\beta}{\alpha} \in (0,1)$ and $B_1 > 0$, while $\Theta\left(\Pi_2\right) = 1$ only for some $\mathcal{E}_2 \in (\alpha+\gamma, \alpha+\beta+\gamma]$. We also obtain:*

$$\lim_{\nu\to 0} m_2 = \begin{cases} 1 & \text{for } \mathcal{E}_2 \in [\gamma, \alpha+\gamma] \\ 1 + \varsigma\Theta\left(\Pi_2\right)\left(\underline{\Pi} - \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right]B_1\right) \geq 1 & \text{for } \mathcal{E}_2 \in (\alpha+\gamma, \alpha+\beta+\gamma]\,. \end{cases} \tag{12}$$

This proposition explains that the premium may be mostly determined by the upper tail of the exchange rate distribution (i.e., large depreciations). The reason is that the premium jumps if the financial constraint binds and pushes m_2 substantially above one, and such an outcome occurs only in the upper uninsurable region.

The details of the argument are as follows. In case (b), the exchange rate distribution is $\Psi = [\gamma, \alpha+\beta+\gamma]$ and the insurable set of exchange rates is $\widetilde{\Psi} = [\beta+\gamma, \alpha+\gamma]$. External lenders' losses are large enough to trigger the financial constraint only after large depreciations, and such depreciations are observed only in the upper uninsurable region. In the limit as $\nu \to 0$, we obtain $m_2 > 1$ *only* if the financial constraint is triggered, so the limit of m_2 is equal to one in both the insurable set and the lower uninsurable region, and above one only for a part of the upper uninsurable region.

An implication is that for this modeling of the external lenders, any monetary policy modification may also be mostly determined by the upper uninsurable region. The premium charged by external lenders is determined mostly by the largest depreciations that they could expect. Accordingly, if the planner internalizes the IS externality and modifies its monetary policy to select lower values of α and β, they would do so mostly to mitigate the largest depreciations rather than to modify other parts of the exchange rate distribution.

4.2 Fiscal Policy

Here we explore two modeling options to explore the influence of fiscal policy on FX market insurability problems. Both modeling options relate to government spending on imports.[17] The first option is that fiscal policy is exogenous and subject to public and private shocks. The second option is that fiscal policy is endogenous and the motivation for public spending is subject to public and private shocks. In the fiscal policy context, private shocks may reflect asymmetric information about political or rent-seeking pressures.

For our first option, we consider a model of exogenous fiscal policy. Government spending on imports in the two periods is G_{Ft} in dollar terms, i.e., a single number at $t = 1$ and a function of both shocks at $t = 2$. They are financed by lump sum taxation, so relative to the model in subsection 2.1, the planner's rebates to households are amended as follows:

$$T_t = \phi W_t N_t + \mu_{t-1} \left(1 + i_{t-1}\right) D_{t-1} - \mathcal{E}_t G_{Ft}.$$

There are no other shocks to tradable endowments or productivity, so we set $Y_{Ft} = \overline{Y}_F$ and $A_t = \overline{A}$ in the households' and firms' maximization problems respectively.

It turns out that the planner problem associated with this model is isomorphic to the planner problem in subsection 3.1.

Proposition 6 (Exogenous fiscal policy). *Suppose that the modeling of fiscal policy follows our first option. The constrained efficient allocation is derived from the planner problem maximizing the utility function (1) subject to the constraints (2)-(4), with Y_{Ft} from that problem replaced by $\overline{Y}_F - G_{Ft}$ and $A_t = \overline{A}$.*

This proposition reveals that fiscal policy can be the underlying source of FX market premia. It can increase the premia through two channels.

First, prior to any shocks, an increase in government spending on imports G_{F1} is identical to a reduction in the initial tradable endowment Y_{F1} and/or an increase in inherited external debt B_0. Accordingly, it tends to increase the external debt B_1 issued at $t = 1$. This debt is subject to any

[17] This subsection builds on the treatment of government spending on imports in Basu and Gopinath (2026) and adds insurability problems. That paper also considers other categories of government spending and financial frictions.

premium that exists on the debt. In addition, the external debt affects the value of the multiplier Λ via equation (6) and the parameters $\{\alpha, \beta, \gamma\}$ in the exchange rate function via equation (8). For $B_1 > 0$, monetary policy may be useful to depreciate away the dollar value of the external debt after adverse external shocks.

Second, the shocks to government spending on imports G_{F2} may be the reason that monetary policy is sensitive to the private shock at $t = 2$ and why a premium exists at $t = 1$. Such shocks generate macroeconomic volatility in that period through the resource constraint (2). Suppose that the private shock θ causes an increase in government spending G_{F2} such that there are less resources available for private import consumption C_{F2}. If so, the planner may set $\beta > 0$ in the exchange rate function (4), because the benefit of depreciating away the dollar value of the external debt becomes higher for larger values of θ. If the country is in case (b), the sensitivity of the exchange rate to the private shock at $t = 2$ generates uninsurability in the FX hedging market and a premium on the FX spot market at $t = 1$.

An implication of this finding is that in our model, premia may arise structurally in countries whose fiscal policy depend on shocks known beforehand only to a subset of FX hedging market participants, and may exhibit episodic surges when such shocks become more important or there is a risk of a regime shift that makes fiscal policy more sensitive to such shocks.

For our second option, we consider a model of endogenous fiscal policy. Government spending on imports and the planner's lump sum rebates follow the same expressions as in our first option. However, we amend the households' welfare to the following:

$$\mathbb{E}\left[\sum_{t=1}^{2} \delta^{t-1} U\left(C_{Ht}, C_{Ft}, G_{Ft}, N_t\right)\right]$$

$$\text{where } U\left(C_{Ht}, C_{Ft}, N_t\right) = \sigma_H \log C_{Ht} + \sigma_F \log C_{Ft} + \sigma_{Gt} \log G_{Ft} - N_t,$$

and the parameter σ_{Gt} is a single number at $t = 1$ and a function of both shocks at $t = 2$.

The planner problem associated with this model is amended relative to the planner problem in subsection 3.1.

Proposition 7 (Endogenous fiscal policy). *Suppose that the modeling of fiscal policy follows our second option. The constrained efficient allocation is derived from the planner problem maximizing the utility function (1) subject to the constraints (2)-(4), with Y_{Ft} from that problem replaced by $\overline{Y}_F - G_{Ft}$, $A_t = \overline{A}$, the planner's period-t utility function V replaced as follows:*

$$V\left(C_{Ft}, \frac{\mathcal{E}_t}{P_H}, G_{Ft}\right) \equiv U\left(\frac{\sigma_H}{\sigma_F}\frac{\mathcal{E}_t}{P_H}C_{Ft}, C_{Ft}, \frac{1}{A_t}\frac{\sigma_H}{\sigma_F}\frac{\mathcal{E}_t}{P_H}C_{Ft}, G_{Ft}\right) \text{ where } \frac{\partial V}{\partial G_{Ft}} = \frac{\sigma_{Gt}}{G_{Ft}},$$

and the planner's choice variables expanded to include $\{G_{F1}, G_{F2}\}$. The allocation satisfies equations (5)-(8) as well as the following condition:

$$G_{F1} = \frac{\sigma_{G1}}{\sigma_F} C_{F1} \text{ and } G_{F2} = \frac{\sigma_{G2}}{\sigma_F} \frac{C_{F2}}{1 + \frac{\sigma_H}{\sigma_F}\tau_{H2}}. \tag{13}$$

This proposition reveals that if fiscal policy is endogenous, the level of government spending on imports is affected by insurability problems in FX markets. Condition (13) comes from combining the FOCs for G_{Ft} and C_{Ft} and establishes how G_{Ft} is determined in both periods.

First, prior to any shocks, the government spending on imports G_{F1} is tied to the private import consumption C_{F1}. As equation (7) indicates, the level of C_{F1} is affected by AD and LC premium externalities. Correspondingly, G_{F1} is also affected by both of these externalities. For example, if C_{F1} is low because the FX market premium is large, G_{F1} is also set to be low. In other words, fiscal discipline in normal times can help make the country ready to manage future shocks.

Second, the sensitivity of government spending on imports G_{F2} to shocks and the parameters in the exchange rate function (4) depend on each other. Equation (8) indicates that if the country is in case (b) and G_{F2} varies with the private shock θ, the parameters $\{\alpha, \beta\}$ are affected by the partial insurability problem because of the AD and IS externalities. Condition (13) indicates that after shocks, the value of $\mathcal{E}_2$ affects the desired value of G_{F2} by affecting the levels of C_{F2} and τ_{H2}.

The planner has the ability to eliminate the insurability problem and the associated premium by constraining fiscal policy to be insensitive to the private shock.

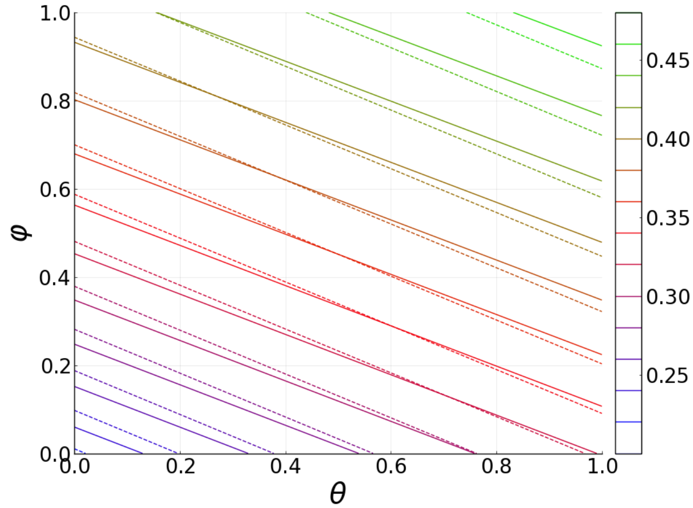
Corollary 3 (Constrained fiscal policy). *Suppose that the modeling of fiscal policy follows our second option but G_{F2} is constrained to be insensitive to the private shock θ while it can remain a function of the public shock φ. Proposition 7 continues to hold except that condition (13) is replaced by the following condition:*

$$G_{F1} = \frac{\sigma_{G1}}{\sigma_F} C_{F1} \text{ and } G_{F2} = \frac{\int_{\theta=0}^{1} \sigma_{G2} \pi\left(\theta|\mathcal{I}_I\right) d\theta}{\int_{\theta=0}^{1} \frac{\sigma_F}{C_{F2}} \left[1 + \frac{\sigma_H}{\sigma_F}\tau_{H2}\right] \pi\left(\theta|\mathcal{I}_I\right) d\theta}. \tag{14}$$

The constrained efficient allocation for subcase (a) (i) features $\beta = 0$, and the premium $\rho = 0$ continues to satisfy the premium constraint (3) after the information sensitivity episode in Lemma 2.

Such a constraint on fiscal policy achieves full insurability in the FX hedging market and eliminates the premium in the FX spot market. In the context of our model of endogenous fiscal policy, the resource constraint (2) indicates that if G_{F2} does not vary with the private shock θ, the feasible levels of C_{F2} are not affected by that shock. As a result, there is no need for the planner to move

Figure 4: How Insurability Problems Change Government Imports G_{F2}



Note: Dashed contours relate to subcase (a) (i) and solid contours relate to case (b). Parameter values are $\sigma_H = 0.5, \sigma_G = 0.5, \delta = 0.96, i^* = \frac{1}{\delta} - 1, \kappa = 0.5, B_0 = 0.5, Y^*_{F2} = 1, \xi = 0.6, \nu = 0.3, \overline{Y}_F = 1,$ and $\overline{A} = 1$.

$\mathcal{E}_2$ with θ. Accordingly, the planner sets $\beta = 0$ and the economy is in subcase (a) (iii). There is no premium on external debt.

Whether such a constraint on fiscal policy is constrained efficient depends on the trade-off between the volatility of σ_{G2} and the level of external debt B_1. On the one hand, welfare is hurt by the constraint on fiscal policy if σ_{G2} is a function of a shock θ that is unknown in advance, i.e., $\pi(\theta|\mathcal{I}_I) = 1$ for all θ, because G_{F2} should ideally vary with θ in this case. On the other hand, welfare is improved by the removal of FX market premia, which is beneficial if B_1 is positive.

Figure 4 shows a simulation for a country where the solution involves the planner reducing but not eliminating the insurability problem. We use the m_2 function from Proposition 4, the modeling option from Proposition 7, and the example function:

$$\sigma_{G2} = \sigma_{G1}\left(1 + \xi\left(\varphi - \frac{1}{2}\right) + \nu\left(\theta - \frac{1}{2}\right)\right). \tag{15}$$

For the parameters listed at the bottom of the figure, we can verify that moving from the full insurability subcase (a) (i) to the partial insurability case (b) causes a higher premium ρ, lower imports C_{F1}, a lower value of $\frac{\beta}{\alpha}$, and lower welfare V^P. The figure shows how such an outcome is achieved. The dashed lines represent the contour map for the levels of G_{F2} as a function of the shocks (φ, θ) in subcase (a) (i) and the solid lines represent the contour map in case (b). The solid lines have a gentler slope than the dashed lines, revealing that the lower value of $\frac{\beta}{\alpha}$ that is needed to reduce the insurability problem is itself facilitated by reducing the sensitivity of G_{F2} to θ.

These results connect the literature on fiscal rules and institutions with the separate international finance literature on FX market premia. Several papers have pointed to fiscal rules and institutions as being beneficial because they impose constraints that reduce public debt (e.g., Wyplosz, 2013; Halac and Yared, 2014; Hatchondo et al., 2022). Our model identifies a novel channel through which such rules and institutions may be beneficial: if they reduce the sensitivity of fiscal policy to private shocks, they can improve the functioning of FX hedging markets and reduce FX spot market premia. Combining Corollaries 2 and 3, we also observe that by reducing these premia, such fiscal policy constraints may reduce the susceptibility of the country to inframarginal return-insensitive investors such as noise traders in FX markets.

4.3 Financial Policy

Here we consider how the desired mix of monetary and financial policies may be affected by the presence of FX market premia. We are interested in macroprudential financial policies, i.e., those which are set at $t = 1$ and generate some associated economic costs, and whose benefits depend on how severe the shocks happen to be at $t = 2$. We model one example of such a policy: we extend our general model to incorporate an FX fund that the planner must set up in advance and can be used to mitigate the output declines caused by the private shock.

Relative to the model in subsection 2.1, we reinterpret the tradable endowment Y_{F2} as the output of export firms who are subject to shocks modeled in reduced form as follows:

$$Y_{F2} = \widetilde{Y}_{F2}(\varphi) + \lambda(\theta, X),$$

where $X \in [0, X_{\max}]$ is an injection of dollar funds by the planner and $\lambda_\theta \leq 0$, $\lambda_X > 0$, $\lambda_{X\theta} > 0$, $\lambda_{XX} < 0$, $\lim_{X \to 0} \mathbb{E}[\lambda_X | \mathcal{I}_I] > (1 + i^*)$, and $\lim_{X \to X_{\max}} \lambda_X < (1 + i^*)$ for all θ. The private financial shock causes a reduction in output but this reduction can be mitigated by dollar injections that are especially valuable for large values of the private shock and have diminishing marginal benefits.

The injections must be financed by the planner borrowing X dollars from abroad and setting up an FX fund at $t = 1$, which is then used to support export firms at $t = 2$. The planner must also repay all the funds with interest in the final period. Since the FX fund is financed by dollar borrowing, its incorporation into the framework does not alter the expression for the local currency external debt D_1 or its dollar value B_1. However, the repayments must be financed by transfers from the households, so the planner's lump sum rebates to households at $t = 2$ are amended as follows:

$$T_2 = \phi W_2 N_2 + \mu_1 (1 + i_1) D_1 - \mathcal{E}_2 X (1 + i^*).$$

The incorporation of the FX fund provides a new macroprudential policy tool to the planner. In the absence of the FX fund, the planner problem associated with this model is a special case of the planner problem in subsection 3.1. In particular, equation (8) indicates that the parameter β in the exchange rate function should not be zero. In other words, since the private financial shock destabilizes the macroeconomy at $t = 2$, the planner should attempt to remedy the problem by setting $\mathcal{E}_2$ to vary with that shock. If the planner can set up an FX fund, the planner problem associated with the extended model is amended relative to the planner problem in subsection 3.1.

Proposition 8 (Financial policy). *Suppose that the modeling of financial policy includes an FX fund. The constrained efficient allocation is derived from the planner problem maximizing the utility function (1) subject to the constraints (2)-(4), with Y_{F2} from that problem replaced by $\widetilde{Y}_{F2}(\varphi) + \lambda(\theta, X) - X(1 + i^*)$, and the planner's choice variables expanded to include X. The allocation satisfies equations (5)-(8) as well as the following condition:*

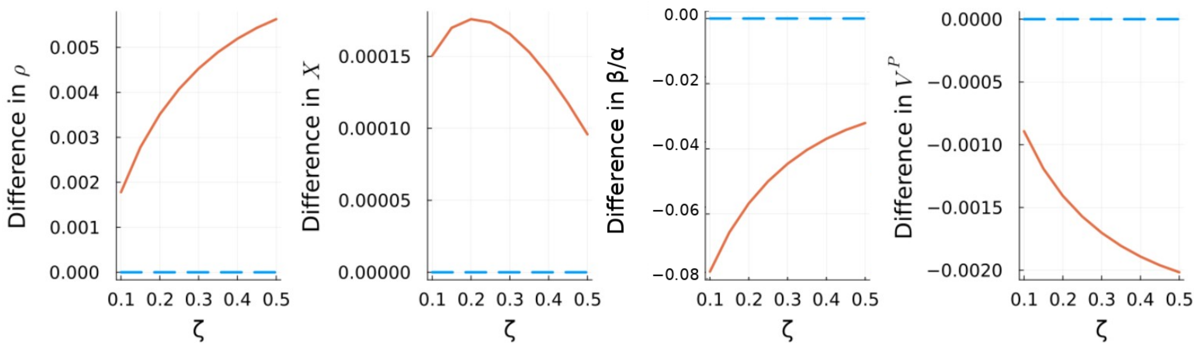
$$(1 + i^*) - \mathbb{E}\left[\lambda_X | \mathcal{I}_I\right] = \frac{Cov\left[\lambda_X, \frac{1+\frac{\sigma_H}{\sigma_F}\tau_{H2}}{C_{F2}} \middle| \mathcal{I}_I\right]}{\mathbb{E}\left[\frac{1+\frac{\sigma_H}{\sigma_F}\tau_{H2}}{C_{F2}} \middle| \mathcal{I}_I\right]}. \tag{16}$$

This proposition establishes how insurability problems affect the desired size of the FX fund. We can use condition (16) to compare subcase (a) (i) and case (b).

For a country in subcase (a) (i), the planner sets $X > 0$ if the private financial shock θ causes a decrease in imports C_{F2} and an increase in the AD wedge τ_{H2}. In such a circumstance, the covariance term on the right hand side of condition (16) is positive, as the shock also increases λ_X. Accordingly, the value of X is increased until the term $\mathbb{E}\left[\lambda_X | \mathcal{I}_I\right]$ on the left hand of the condition decreases sufficiently below $(1 + i^*)$.

For a country in case (b), the planner generally selects a different value of X, and may select a higher value. Proposition 3 establishes that the planner selects a different value of the ratio $\frac{\beta}{\alpha}$ in case (b) than in subcase (a) (i), except in knife-edge cases. This change in the exchange rate function causes changes in C_{F2} and τ_{H2}, which in turn alter the desired value of X according to condition (16). Moreover, as in Corollary 1, the planner may select a lower value of $\frac{\beta}{\alpha}$ in case (b) than in subcase (a) (i), i.e., they may reduce the sensitivity of the exchange rate to the private shock to reduce FX market premia. The side-effect of this modified monetary policy may be that the private financial shock θ causes a larger decrease in C_{F2} and increase in τ_{H2}. The covariance term on the right hand side of condition (16) may then be higher. If so, it may result in the planner selecting a larger value of X to reduce the term $\mathbb{E}\left[\lambda_X | \mathcal{I}_I\right]$ further on the left hand side of the condition.

Figure 5: How Insurability Problems Affect Financial Policy, Given the Value of ζ



Note: Difference in variable = Constrained efficient value in case (b) minus value in subcase (a) (i). Parameter values are $\sigma_H = 0.5, \delta = 0.96, i^* = \frac{1}{\delta} - 1, \kappa = 0.5, B_0 = 0.5, Y^*_{F2} = 1, \xi = 0.6, \nu = 0.3,$ and $\overline{A} = 1$.

Figure 5 shows a simulation for a country where the planner indeed selects a higher value of X. We use the m_2 function from Proposition 4, the example functions from equation (11), and the following additional example function:

$$\lambda(\theta, X) = -\nu\left(\theta\left(1 - X^{\varsigma}\right) - \frac{1}{2}\right) \text{ where } X_{\max} = 1. \tag{17}$$

The red lines in the figure plot the difference for the constrained efficient variables going from the full insurability subcase (a) (i) to the partial insurability case (b). Insurability problems cause a higher premium ρ, a larger FX fund X, a lower value of $\frac{\beta}{\alpha}$, and lower welfare V^P.

This finding has implications for the desired mix of monetary and financial policies in our model. If insurability problems mean that the constrained efficient monetary policy features a low sensitivity of the exchange rate to the private shock, financial policies may have to play a greater role to address that shock. Indeed, if financial policies can directly mitigate the macroeconomic impact of the private shock, there is less need for the exchange rate to vary with the private shock, which means that the premia would be lower. Relative to papers which study the integrated policy mix in the context of symmetric information (e.g., Farhi and Werning, 2016; Bianchi and Lorenzoni, 2022; Basu et al., 2025), our model suggests that there may be a greater role for the kinds of financial policies which can reduce any asymmetric information about financial conditions among FX market participants. This recommendation would be reversed for the kinds of financial policies which may exacerbate asymmetric information.

4.4 Institutions to Manage Information

Finally, we use our model to explore whether outcomes can be improved by setting up institutions to limit asymmetric information.

We add a new period $t = 0$ which occurs before the two periods of the model in subsection 2.1. There is no discounting between $t = 0$ and $t = 1$. In the new period, no shocks are known to any agents, and the planner must commit to an institutional regime $\Delta \in \{N, S, T\}$.

- $\Delta = N$ corresponds to the planner committing at $t = 0$ to do nothing at $t = 1$, so the new period has no impact on the model. We assume that if the planner does nothing, the country is in case (b) at $t = 1$, i.e., $\mathcal{I}_I = \mathcal{I}_\emptyset$, $\mathcal{I}_S = \mathcal{I}_\theta$, and $\beta > 0$.

- $\Delta = S$ corresponds to the planner committing at $t = 0$ to keep secret all information related to the private shock at $t = 1$. We assume that the planner can maintain secrecy at a cost $\Upsilon^S \geq 0$. If they do so, they convert the country to subcase (a) (i) at $t = 1$, i.e., $\mathcal{I}_I = \mathcal{I}_S = \mathcal{I}_\emptyset$.

- $\Delta = T$ corresponds to the planner committing at $t = 0$ to collect and publish all information related to the private shock at $t = 1$. We assume that the planner can collect the information at a cost $\Upsilon^T \geq 0$. If they do so, they set $\{\alpha, \beta, \gamma\}$ at $t = 0$ and convert the country to subcase (a) (ii) at $t = 1$, i.e., $\mathcal{I}_I = \mathcal{I}_S = \mathcal{I}_\theta$.

Out of this set of regimes, we observe that while the options of secrecy S and publication T appear diametrically opposed to each other, both of them eliminate the premium by achieving symmetric information at $t = 1$.

The following result relates the planner's decision at $t = 0$ to the different possible versions of the planner problem in subsection 3.1.

Proposition 9 (Institutional regime). *Suppose that the planner commits to an institutional regime at $t = 0$. The planner's decision at $t = 0$ solves:*

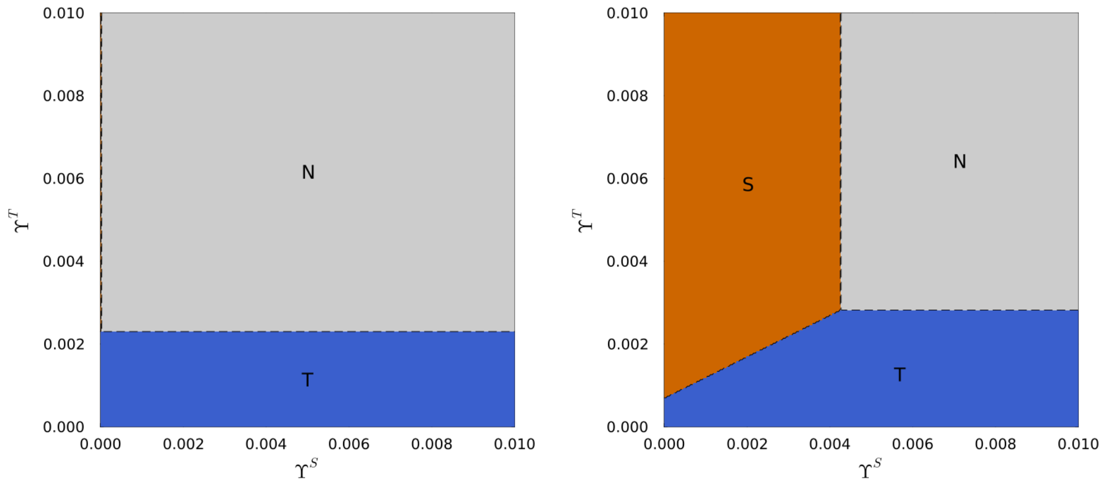
$$V^{P0} = \max_{\Delta} \left\{ V^{\Delta} \right\}$$

$$\text{where } V^N = V^{P,(b)},\ V^S = V^{P,(a)(i)} - \Upsilon^S, \text{ and } V^T = \int_{\theta=0}^{1} V^{P,(a)(ii)} d\theta - \Upsilon^T,$$

and $V^{P,(x)}$ denotes the welfare from maximizing the utility function (1) subject to the constraints (2)-(4) for a country in case (x) at $t = 1$ and the additional constraint that in subcase (a) (ii), the planner sets $\{\alpha, \beta, \gamma\}$ at $t = 0$.

Figure 6: How Institutional Design Varies with Inherited Debt B_0

(A) $B_0 = 0$ **(B)** $B_0 = 0.5$



Note: N = Do nothing, S = Secrecy, and T = Publication. Parameter values are $\sigma_H = 0.5, \delta = 0.96, i^* = \frac{1}{\delta} - 1, \kappa = 0.5, Y^*_{F2} = 1, \xi = 0.6, \nu = 0.3,$ and $\overline{A} = 1$.

This proposition captures that in our model, the planner's choice of institutional regime should weigh the benefits of reducing FX market premia against the costs of collecting information or keeping it secret.

The planner chooses institutions at $t = 0$ to modify the information structure at $t = 1$ only if the benefits outweigh the costs. If the benefit of reducing the premium is small, while the costs of both of the regimes of secrecy and publication are large, the planner may commit to do nothing. However, if the benefit of reducing the premium is large because the inherited external debt B_0 is large and the IS externality is severe, while the costs of either of the regimes of secrecy or publication are small, the planner may commit to one of these two regimes instead of doing nothing.

Figure 6 shows simulations which confirm this reasoning. We use the m_2 function from Proposition 4 and the example functions from equation (11), evaluated at the parameters listed at the bottom of the figure. Panel A shows that for low inherited external debt $B_0 = 0$, the planner chooses $\Delta = T$ for low values of Υ^T and $\Delta = N$ otherwise. By contrast, panel B shows that for the higher inherited external debt $B_0 = 0.5$, the planner chooses $\Delta \in \{S, T\}$ when at least one of the costs $\{\Upsilon^S, \Upsilon^T\}$ are small, and chooses $\Delta = N$ only if both of the costs are large.

The choice between the secrecy and publication regimes generally depends on the kind of information. For information which is difficult to collect and publish but easy to keep secret, i.e., Υ^T is high and Υ^S is low, a commitment to secrecy and keeping all agents uninformed is the least costly way of eliminating premia. For information which is easy to collect and publish but difficult to keep secret, i.e., Υ^T is low and Υ^S is large, a commitment to publication and keeping all agents informed is the least costly way of converting the country from asymmetric to symmetric information and eliminating premia.

Some kinds of information may switch between the two categories. For example, some economic data may be difficult to collect within the first week but can be inferred by a large set of private sector agents after a month. In this context, an environment of symmetric information could be preserved if institutions keep the collection of sensitive economic data secret from all agents to begin with, and then release it in a public fashion once it is collected. Such an approach could minimize the likelihood of any episodes of asymmetric information which may cause disruptions and premia in FX markets.

5 Conclusion

We propose a tractable framework that relates UIP premia to the insurability of exchange rates, and then explains how macroeconomic policies affect that insurability via a novel information sensitivity externality. In our model, insurability problems endogenously arise from asymmetric information about the tails of the exchange rate distribution, so the constrained efficient policies seek to reduce the sensitivity of the exchange rate to private information.

We illustrate the adaptability of our general model by exploring four applications: the composition of external lenders, fiscal policy, financial policy, and institutions to limit asymmetric information. Of course, the insights from these applications may interact. One example is that if fiscal policy is the underlying source of insurability problems, the design of fiscal institutions could be related to the composition of external lenders. Another example is that the use of financial policies may be especially beneficial if the country's external lenders require large premia to cover the risk of exchange rate depreciations.

We leave several topics for future research. On the technical dimension, the exchange rate can be allowed to be a nonlinear function of the shocks, which would change the shape of the insurable set and the uninsurable regions with some cost to tractability. On the practical dimension, we need to better understand how premia in FX markets can be affected by additional complications such as time consistency and poor coordination across policy tools, as well as how the problem of asymmetric information may vary across countries and tools.

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APPENDIX

A Proofs of Results in Section 2

Proof of Proposition 1

As indicated in the main text, we establish the proposition by solving the FX hedging market backward. For each $\mathcal{E}_2 \in \Psi$, we suppose that the insurers for this exchange rate decide $d = 1$ such that $\mathcal{E}_2 \in \widetilde{\Psi}$, and then we establish whether there is a contradiction and the insurers prefer $d = 0$ instead.

In step 2, speculators have the demands $Q_S(\mathcal{E}_2)$ shown on page 11 of the main text. External lenders have the aggregate demands $Q_L(\mathcal{E}_2)$ derived from the expressions on page 9 of the main text. They do not observe any information that can help them update their information set $\mathcal{I}_L = \mathcal{I}_I$, and their profits do not depend on the speculators' demands. Moreover, changes in the price $p(\mathcal{E}_2)$ for a specific exchange rate $\mathcal{E}_2$ do not affect $\overline{M}$. For $\pi(\mathcal{E}_2|\mathcal{I}_L) > 0$, we obtain the following relations for the lenders' aggregate decisions:

$$\frac{dQ_L(\mathcal{E}_2)}{dp(\mathcal{E}_2)} < 0 \text{ and } \lim_{p(\mathcal{E}_2)\to 0} Q_L(\mathcal{E}_2) = \infty,$$

with $Q_L(\mathcal{E}_2)$ being finite for $p(\mathcal{E}_2) > 0$. For $\pi(\mathcal{E}_2|\mathcal{I}_L) \to 0$, we obtain that $Q_L(\mathcal{E}_2)$ is indeterminate if $p(\mathcal{E}_2) \to 0$, while $Q_L(\mathcal{E}_2) \to Q_{\min}(\mathcal{E}_2)$ if $p(\mathcal{E}_2) > 0$.

In step 1, there are two cases, which we next analyze in turn. For each of the cases, the insurers' problems are shown on page 12 of the main text. The insurers' profits depend on the demands of the lenders and the speculators, so the insurers must game out how these other agents respond to different prices $p(\mathcal{E}_2)$.

The first case is that $\mathcal{I}_I = \mathcal{I}_S \in \{\mathcal{I}_\emptyset, \mathcal{I}_\theta\}$.

- For $\pi(\mathcal{E}_2|\mathcal{I}_I) = \pi(\mathcal{E}_2|\mathcal{I}_S) > 0$, the insurers do not select $p(\mathcal{E}_2) = 0$ as that choice would result in losses owing to $Q_L(\mathcal{E}_2) \to \infty$ and $Q_S(\mathcal{E}_2) \to \infty$. If the insurers select some $p(\mathcal{E}_2) > 0$, the argument in the main text applies, establishing that $(1 + i^*)\, p(\mathcal{E}_2) = \pi(\mathcal{E}_2|\mathcal{I}_I)$.

- For $\pi(\mathcal{E}_2|\mathcal{I}_I) = \pi(\mathcal{E}_2|\mathcal{I}_S) = 0$, the insurers can avoid losses by selecting $p(\mathcal{E}_2) = 0$, establishing that $(1 + i^*)\, p(\mathcal{E}_2) = \pi(\mathcal{E}_2|\mathcal{I}_I)$. They do not select some $p(\mathcal{E}_2) > 0$ as that choice would result in losses owing to $Q_L(\mathcal{E}_2) \to Q_{\min}(\mathcal{E}_2)$ and $Q_S(\mathcal{E}_2) \to -\infty$.

The second case is that $\mathcal{I}_I = \mathcal{I}_\emptyset$ and $\mathcal{I}_S = \mathcal{I}_\theta$.

- For $\pi(\mathcal{E}_2|\mathcal{I}_I) > 0$, the insurers do not select $p(\mathcal{E}_2) = 0$ as that choice would result in losses. We would obtain $Q_L(\mathcal{E}_2) \to \infty$ and it is possible for $\pi(\mathcal{E}_2|\mathcal{I}_S) \geq (1 + i^*)\, p(\mathcal{E}_2)$. If $\pi(\mathcal{E}_2|\mathcal{I}_S) > (1 + i^*)\, p(\mathcal{E}_2)$, there would be $Q_S(\mathcal{E}_2) \to \infty$, resulting in losses for the insurers. If $\pi(\mathcal{E}_2|\mathcal{I}_S) = (1 + i^*)\, p(\mathcal{E}_2)$, there would be $Q_S(\mathcal{E}_2) \in \mathbb{R}$, resulting in zero profits. Adapting the argument in the main text for this price $p(\mathcal{E}_2) = 0$, the insurers would prefer $d = 0$ as they cannot ensure that the condition $(1 + i^*)\, p(\mathcal{E}_2) = \pi(\mathcal{E}_2|\mathcal{I}_S)$ holds with probability one. If the insurers select some $p(\mathcal{E}_2) > 0$, the argument in the main text applies, establishing that the insurers decide $d = 1$ only if $\pi(\mathcal{E}_2|\mathcal{I}_I) = \pi(\mathcal{E}_2|\mathcal{I}_S)$. In that case, we obtain $(1 + i^*)\, p(\mathcal{E}_2) = \pi(\mathcal{E}_2|\mathcal{I}_I)$.

- For $\pi(\mathcal{E}_2|\mathcal{I}_I) = 0$, it follows by definition that $\pi(\mathcal{E}_2|\mathcal{I}_S) = 0$ for all θ, which in turn means that $\pi(\mathcal{E}_2|\mathcal{I}_I) = \pi(\mathcal{E}_2|\mathcal{I}_S) = 0$. The insurers select $p(\mathcal{E}_2) = 0$. For this price, we would obtain an indeterminate $Q_L(\mathcal{E}_2)$ and it must be that $\pi(\mathcal{E}_2|\mathcal{I}_S) = (1 + i^*)\, p(\mathcal{E}_2)$. There would be $Q_S(\mathcal{E}_2) \in \mathbb{R}$, resulting in zero profits for the insurers. Adapting the argument in the main text, the insurers decide $d = 1$ and we establish that $(1 + i^*)\, p(\mathcal{E}_2) = \pi(\mathcal{E}_2|\mathcal{I}_I)$. The insurers do not select some $p(\mathcal{E}_2) > 0$ as that choice would result in losses. We would obtain $Q_L(\mathcal{E}_2) \to Q_{\min}(\mathcal{E}_2)$ and then it must be that $\pi(\mathcal{E}_2|\mathcal{I}_S) < (1 + i^*)\, p(\mathcal{E}_2)$. There would be $Q_S(\mathcal{E}_2) \to -\infty$, resulting in losses for the insurers.

These arguments establish the results in the proposition. ■

Proof of Proposition 2

For $\alpha \geq \beta \geq 0$, the set of exchange rates Ψ can be partitioned into three regions as in Figure 1. Substituting $\mathcal{I}_L = \mathcal{I}_I$ and rearranging the expression for the external lenders' bond position from page 9 of the main text, we can obtain the following condition with three terms, each representing one of the regions:

$$\begin{aligned} 0 = & \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} M_2 \left[\eta_2 - (1+i^*)\right] \pi\left(\theta|\mathcal{I}_I\right) d\varphi d\theta \\ & + \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} M_2 \left[\eta_2 - (1+i^*)\right] \pi\left(\theta|\mathcal{I}_I\right) d\varphi d\theta \\ & + \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} M_2 \left[\eta_2 - (1+i^*)\right] \pi\left(\theta|\mathcal{I}_I\right) d\varphi d\theta. \end{aligned}$$

The probability density functions of the exchange rates $\mathcal{E}_2$ in these three respective regions depend on the information sets:

$$\pi\left(\mathcal{E}_2|\mathcal{I}_\theta\right) = \begin{cases} \frac{1}{\alpha} & \text{for } \mathcal{E}_2 \in [\beta+\gamma, \alpha+\gamma] \\ 0 \text{ if } \mathcal{E}_2 < \beta\theta+\gamma; \frac{1}{\alpha} \text{ if } \mathcal{E}_2 \geq \beta\theta+\gamma & \text{for } \mathcal{E}_2 \in [\gamma, \beta+\gamma) \\ \frac{1}{\alpha} \text{ if } \mathcal{E}_2 \leq \alpha+\beta\theta+\gamma; 0 \text{ if } \mathcal{E}_2 > \alpha+\beta\theta+\gamma & \text{for } \mathcal{E}_2 \in (\alpha+\gamma, \alpha+\beta+\gamma] \end{cases}$$

$$\pi\left(\mathcal{E}_2|\mathcal{I}_\emptyset\right) = \begin{cases} \int_{\theta=0}^{1} \frac{1}{\alpha} d\theta = \frac{1}{\alpha} & \text{for } \mathcal{E}_2 \in [\beta+\gamma, \alpha+\gamma] \\ \int_{\theta=0}^{\frac{\mathcal{E}_2-\gamma}{\beta}} \frac{1}{\alpha} d\theta = \frac{\mathcal{E}_2-\gamma}{\alpha\beta} & \text{for } \mathcal{E}_2 \in [\gamma, \beta+\gamma) \\ \int_{\theta=\frac{\mathcal{E}_2-\alpha-\gamma}{\beta}}^{1} \frac{1}{\alpha} d\theta = \frac{\alpha+\beta+\gamma-\mathcal{E}_2}{\alpha\beta} & \text{for } \mathcal{E}_2 \in (\alpha+\gamma, \alpha+\beta+\gamma]. \end{cases}$$

Next, we consider the following cases. Note that in all the cases, $Q_I\left(\mathcal{E}_2\right) = Q_S\left(\mathcal{E}_2\right) + \psi Q_L\left(\mathcal{E}_2\right)$.

Subcase (a) (i). $\mathcal{I}_I = \mathcal{I}_S = \mathcal{I}_\emptyset$. Since $\mathcal{I}_I = \mathcal{I}_\emptyset$, $\pi\left(\theta|\mathcal{I}_I\right) = 1$. Since $\mathcal{I}_I = \mathcal{I}_S$, $\pi\left(\mathcal{E}_2|\mathcal{I}_I\right) = \pi\left(\mathcal{E}_2|\mathcal{I}_S\right)$ for all $\mathcal{E}_2 \in \Psi$ irrespective of θ. Then Proposition 1 indicates that all exchange rates are insurable, i.e., $\widetilde{\Psi} = \Psi$. In addition, $(1+i^*)\, p\left(\mathcal{E}_2\right) = \pi\left(\mathcal{E}_2|\mathcal{I}_I\right)$ within the insurable set. Correspondingly, $Q_L\left(\mathcal{E}_2\right)$ is set for all $\mathcal{E}_2 \in \Psi$ such that $M_2 = \overline{M}$ in all the terms in the above condition for the bond position, while $Q_S\left(\mathcal{E}_2\right) \in \mathbb{R}$. Dividing that condition through by $\overline{M}$, we establish $\mathbb{E}\left[\eta_2\right] = (1+i^*)$ and all the results for this subcase of the proposition.

Subcase (a) (ii). $\mathcal{I}_I = \mathcal{I}_S = \mathcal{I}_\theta$. Since $\mathcal{I}_I = \mathcal{I}_\theta$, $\pi\left(\theta|\mathcal{I}_I\right) = 1$ only at the known value of θ and $= 0$ elsewhere. Since $\mathcal{I}_I = \mathcal{I}_S$, $\pi\left(\mathcal{E}_2|\mathcal{I}_I\right) = \pi\left(\mathcal{E}_2|\mathcal{I}_S\right)$ for all $\mathcal{E}_2 \in \Psi$ irrespective of θ. Then Proposition 1 indicates that all exchange rates are insurable, i.e., $\widetilde{\Psi} = \Psi$. In addition, $(1+i^*)\, p\left(\mathcal{E}_2\right) = \pi\left(\mathcal{E}_2|\mathcal{I}_I\right)$ within the insurable set. Correspondingly, $Q_L\left(\mathcal{E}_2\right)$ is set for all $\mathcal{E}_2 \in \Psi$ such that $M_2 = \overline{M}$ in all the terms in the above condition for the bond position, while $Q_S\left(\mathcal{E}_2\right) \in \mathbb{R}$. Dividing that condition through by $\overline{M}$, we establish $\mathbb{E}\left[\eta_2\right] = (1+i^*)$ and all the results for this subcase of the proposition.

Subcase (a) (iii). $\mathcal{I}_I = \mathcal{I}_\emptyset$, $\mathcal{I}_S = \mathcal{I}_\theta$, and $\beta = 0$. Since $\mathcal{I}_I = \mathcal{I}_\emptyset$, $\pi\left(\theta|\mathcal{I}_I\right) = 1$. Since $\beta = 0$, $\pi\left(\mathcal{E}_2|\mathcal{I}_I\right) = \pi\left(\mathcal{E}_2|\mathcal{I}_S\right)$ for all $\mathcal{E}_2 \in \Psi$ irrespective of θ even though $\mathcal{I}_S \supset \mathcal{I}_I$. The reason is that the second and third terms in the above condition for the bond position are zero and there are no exchange rates within the regions represented by those terms. Then Proposition 1 indicates that all exchange rates are insurable, i.e., $\widetilde{\Psi} = \Psi$. In addition, $\pi\left(\mathcal{E}_2|\mathcal{I}_I\right) = (1+i^*)\, p\left(\mathcal{E}_2\right)$ within the insurable set. Correspondingly, $Q_L\left(\mathcal{E}_2\right)$ is set for all $\mathcal{E}_2 \in \Psi$ such that $M_2 = \overline{M}$ in the first and only remaining term in the above condition for the bond position, while $Q_S\left(\mathcal{E}_2\right) \in \mathbb{R}$. Dividing that condition through by $\overline{M}$, we establish $\mathbb{E}\left[\eta_2\right] = (1+i^*)$ and all the results for this subcase of the proposition.

Case (b). $\mathcal{I}_I = \mathcal{I}_\emptyset$, $\mathcal{I}_S = \mathcal{I}_\theta$, and $\beta > 0$. Since $\mathcal{I}_I = \mathcal{I}_\emptyset$, $\pi\left(\theta|\mathcal{I}_I\right) = 1$. Then Proposition 1 indicates that only the exchange rates $\mathcal{E}_2 \in [\beta+\gamma, \alpha+\gamma]$ are insurable, i.e., $\widetilde{\Psi} = [\beta+\gamma, \alpha+\gamma]$. The probability of $\mathcal{E}_2 \in \widetilde{\Psi}$ is $1-\frac{\beta}{\alpha}$. In addition, $(1+i^*)\, p\left(\mathcal{E}_2\right) = \pi\left(\mathcal{E}_2|\mathcal{I}_I\right)$ for all $\mathcal{E}_2 \in \widetilde{\Psi}$. Correspondingly, $Q_L\left(\mathcal{E}_2\right)$ is set for all $\mathcal{E}_2 \in \widetilde{\Psi}$ such that $M_2 = \overline{M}$ in the first term in the above condition for the bond position, while $Q_S\left(\mathcal{E}_2\right) \in \mathbb{R}$. Meanwhile, $p\left(\mathcal{E}_2\right) = \emptyset$ and $Q_L\left(\mathcal{E}_2\right) = Q_S\left(\mathcal{E}_2\right) = 0$ for all $\mathcal{E}_2 \notin \widetilde{\Psi}$. Dividing the condition for the bond position through by

$\overline{M}$, we establish the version of the condition shown in the proposition. Applying the definition of the covariance, we obtain the expression for the premium ρ. Finally, $m_2 = m_2\left(B_1, \eta_2, \{\eta_2\}_{\mathcal{E}_2 \in \widetilde{\Psi}}, \frac{\beta}{\alpha}\right)$ for all $\mathcal{E}_2 \notin \widetilde{\Psi}$ because M_2 depends on the variables $\{B_1, \eta_2\}$ outside the insurable set $\widetilde{\Psi}$, while $\overline{M}$ depends on the variables $\{B_1, \eta_2\}$ within the insurable set $\widetilde{\Psi}$ and the boundaries of that set. ■

B Planner Problem

Here we explain how to extract the details of the constrained efficient allocation. Equations (1)-(4) characterize that allocation in terms of the planner's choice variables $\{C_{F1}, C_{F2}, \mathcal{E}_1, \alpha, \beta, \gamma, \chi, P_H\}$. The remaining variables listed in Definitions 1 and 2 can be derived as follows:

$$i_1 = \frac{1}{\delta \mathcal{E}_1 C_{F1} \mathbb{E}\left[\frac{1}{\mathcal{E}_2 C_{F2}}\right]} - 1$$

$$\mu_1 = 1 - \delta \chi C_{F1} \mathbb{E}\left[\frac{1}{\mathcal{E}_2 C_{F2}}\right]$$

$$\phi = \sigma_F P_H \frac{\varepsilon - 1}{\varepsilon} \frac{1+\delta}{\frac{\mathcal{E}_1 C_{F1}}{A_1} + \delta \mathbb{E}\left[\frac{\mathcal{E}_2 C_{F2}}{A_2}\right]} - 1$$

$$\mathcal{E}_2 = \alpha\varphi + \beta\theta + \gamma$$

$$C_{Ht} = Y_{Ht} = \frac{\sigma_H}{\sigma_F}\frac{\mathcal{E}_t C_{Ft}}{P_H},\ N_t = \frac{Y_{Ht}}{A_t}, \text{ and } W_t = \frac{1}{\sigma_F}\mathcal{E}_t C_{Ft} \text{ for } t \in \{1,2\}$$

$$D_2 = \left[B_0\left(1+i^*\right) + \left(C_{F1} - Y_{F1}\right)\right]\mathcal{E}_1,$$

while the quantities $\{Q_L(\mathcal{E}_2), Q_I(\mathcal{E}_2), Q_S(\mathcal{E}_2)\}_{\mathcal{E}_2 \in \Psi}$, prices $\{p(\mathcal{E}_2)\}_{\mathcal{E}_2 \in \Psi}$, exchange rate sets $\left\{\Psi, \widetilde{\Psi}\right\}$ and probability distributions $\{\pi(\mathcal{E}_2|\mathcal{I}_I), \pi(\mathcal{E}_2|\mathcal{I}_S)\}_{\mathcal{E}_2 \in \Psi}$ pertaining to the FX hedging market follow the expressions in the proof of Proposition 2.

Next, we complement the analysis in subsection 3.2 with the FOCs of the planner problem and additional expressions which are not provided there. We assume that the solution satisfies $\alpha > \beta \geq 0$. The FOC with respect to $\mathcal{E}_1$ is provided on page 18 of the main text. The FOC with respect to χ is as follows:

$$\frac{B_1}{\chi}\int_{\theta=0}^{1}\int_{\varphi=0}^{1}\Phi(\varphi,\theta)\,\pi(\theta|\mathcal{I}_I)\,d\varphi d\theta = \Lambda\left[\int_{\theta=0}^{1}\int_{\varphi=0}^{1}\frac{1}{\mathcal{E}_2}\pi(\theta|\mathcal{I}_I)\,d\varphi d\theta + z_\chi\right],$$

where $\Phi(\varphi,\theta)\,\pi(\theta|\mathcal{I}_I)$ is the multiplier on the resource constraint (2) for the state of nature (φ,θ). The FOCs with respect to C_{F1} and C_{F2} are as follows:

$$\frac{\sigma_F}{C_{F1}}\left[1 + \frac{\sigma_H}{\sigma_F}\tau_{H1}\right] = \int_{\theta=0}^{1}\int_{\varphi=0}^{1}\Phi(\varphi,\theta)\,\pi(\theta|\mathcal{I}_I)\,d\varphi d\theta + \Lambda z_{F1}$$

$$\frac{\delta\sigma_F}{C_{F2}}\left[1 + \frac{\sigma_H}{\sigma_F}\tau_{H2}\right] = \Phi(\varphi,\theta)\frac{\mathcal{E}_2}{\chi}.$$

The FOCs with respect to each parameter $\omega \in \{\alpha, \beta, \gamma\}$ are as follows:

$$\int_{\theta=0}^{1}\int_{\varphi=0}^{1}\frac{\partial \mathcal{E}_2}{\partial\omega}\left\{\frac{1}{\mathcal{E}_2}\delta\sigma_H\tau_{H2} + B_1\frac{1}{\mathcal{E}_2}\Phi(\varphi,\theta)\right\}\pi(\theta|\mathcal{I}_I)\,d\varphi d\theta$$
$$= \Lambda\left[\int_{\theta=0}^{1}\int_{\varphi=0}^{1}\frac{\partial\mathcal{E}_2}{\partial\omega}\frac{\chi}{(\mathcal{E}_2)^2}\pi(\theta|\mathcal{I}_I)\,d\varphi d\theta + z_{\omega 1} + z_{\omega 2}\right].$$

The FOC with respect to P_H is as follows:

$$\tau_{H1} + \delta \int_{\theta=0}^{1} \int_{\varphi=0}^{1} \tau_{H2} \pi \left(\theta | \mathcal{I}_I\right) d\varphi d\theta = 0.$$

This FOC is redundant given the FOCs with respect to $\{\mathcal{E}_1, \alpha, \beta, \gamma, \chi\}$, provided that the following condition holds:

$$\frac{\partial m_2}{\partial \alpha}\alpha + \frac{\partial m_2}{\partial \beta}\beta + \frac{\partial m_2}{\partial \gamma}\gamma + \frac{\partial m_2}{\partial \chi}\chi = 0.$$

This condition does indeed hold because $m_2 = m_2\left(B_1, \eta_2, \{\eta_2\}_{\mathcal{E}_2 \in \widetilde{\Psi}}, \frac{\beta}{\alpha}\right)$ and we can establish that $\frac{\partial \eta_2}{\partial \alpha}\alpha + \frac{\partial \eta_2}{\partial \beta}\beta + \frac{\partial \eta_2}{\partial \gamma}\gamma + \frac{\partial \eta_2}{\partial \chi}\chi = 0$ and $\frac{\partial\left(\frac{\beta}{\alpha}\right)}{\partial \alpha}\alpha + \frac{\partial\left(\frac{\beta}{\alpha}\right)}{\partial \beta}\beta = 0$.

Next, we turn to how to amend the FOCs when $\alpha > \beta \geq 0$ is not satisfied at an interior solution. We consider two scenarios. In the first scenario, we suppose that $\alpha = 0$ or $\beta = 0$ are binding. If so, we replace the equality with the inequality $<$ in the FOC with respect to that parameter. In the second scenario, we suppose that $\beta > \alpha \geq 0$. If so, we make more substantial modifications to the analysis. We amend the premium constraint (3) in Proposition 2 and the planner problem as follows:

$$0 = \int_{\theta=0}^{1} \int_{\varphi=0}^{1} m_2 \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] \pi \left(\theta | \mathcal{I}_I\right) d\varphi d\theta, \tag{B.1}$$

as there is no longer a shaded region in Figure 1. In subcases (a) (i) and (ii), we continue to obtain $m_2 = 1$, while subcase (a) (iii) cannot occur when $\beta > 0$. In case (b), the probability of $\mathcal{E}_2 \in \widetilde{\Psi}$ is 0, the insurable set is $\widetilde{\Psi} = \emptyset$, and there are no longer any upper or lower insurability contours. In the FOCs, we amend the following definitions:

$$z_\chi = \int_{\theta=0}^{1} \int_{\varphi=0}^{1} \left\{ [m_2 - 1] \frac{1}{\mathcal{E}_2} + \frac{\partial m_2}{\partial \chi} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] \right\} \pi \left(\theta | \mathcal{I}_I\right) d\varphi d\theta$$

$$z_{F1} = -\int_{\theta=0}^{1} \int_{\varphi=0}^{1} \frac{\partial m_2}{\partial B_1} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] \pi \left(\theta | \mathcal{I}_I\right) d\varphi d\theta$$

$$z_{\omega 1} = \int_{\theta=0}^{1} \int_{\varphi=0}^{1} \left\{ [m_2 - 1] \frac{\partial \mathcal{E}_2}{\partial \omega} \frac{\chi}{(\mathcal{E}_2)^2} - \frac{\partial m_2}{\partial \omega} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] \right\} \pi \left(\theta | \mathcal{I}_I\right) d\varphi d\theta$$

$$z_{\omega 2} = 0.$$

Finally, for comparison purposes, we consider how the analysis changes if the model is amended such that there are no premia and the exchange rate function is unrestricted. Specifically, we consider an amended planner problem where we set $m_2 = 1$ in all the terms of the premium constraint (3), we remove the restricted exchange rate function (4), and we permit optimization over $\mathcal{E}_2$ instead of $\{\alpha, \beta, \gamma\}$. There are two modifications to the analysis. First, since $m_2 = 1$ and $\frac{\partial m_2}{\partial \chi} = \frac{\partial m_2}{\partial B_1} = 0$, we obtain $z_\chi = z_{F1} = 0$ in the FOCs with respect to $\{C_{F1}, \chi\}$ and in equations (6)-(7). Second, we replace the FOCs with respect to $\omega \in \{\alpha, \beta, \gamma\}$ with new FOCs with respect to $\mathcal{E}_2$. For each state of nature (φ, θ), we obtain:

$$\frac{1}{\mathcal{E}_2}\delta \sigma_H \tau_{H2} + B_1 \frac{1}{\mathcal{E}_2} \Phi\left(\varphi, \theta\right) = \Lambda \frac{\chi}{(\mathcal{E}_2)^2}.$$

Correspondingly, we replace equation (8) with the following equation:

$$\delta \sigma_H \tau_{H2} + B_1 \frac{\chi}{\mathcal{E}_2} \frac{\delta \sigma_F}{C_{F2}} \left[1 + \frac{\sigma_H}{\sigma_F}\tau_{H2}\right] = \Lambda \frac{\chi}{\mathcal{E}_2}. \tag{B.2}$$

C Proofs of Results in Sections 3-4

Proof of Lemma 1

For all the subcases within case (a) from Proposition 2, we obtain $m_2 = 1$ for every term in the premium constraint (3) which contains m_2. For subcases (a) (i) and (a) (ii), we use the conditions $m_2 = 1$ and $\frac{\partial m_2}{\partial \chi} = \frac{\partial m_2}{\partial B_1} = \frac{\partial m_2}{\partial \omega} = 0$ for all $\omega \in \{\alpha, \beta, \gamma\}$ to obtain $z_\chi = z_{F1} = z_{\omega 1} = z_{\omega 2} = 0$. For subcase (a) (iii), we use the condition $\beta = 0$ to obtain $z_\chi = z_{F1} = z_{\omega 1} = 0$ for all $\omega \in \{\alpha, \beta, \gamma\}$ and $z_{\omega 2} = 0$ for all $\omega \in \{\alpha, \gamma\}$. ■

Proof of Lemma 2

The country starts with $\mathcal{I}_I = \mathcal{I}_S \in \{\mathcal{I}_\emptyset, \mathcal{I}_\theta\}$, so it may be in subcase (a) (i) or (ii). Given Proposition 2, the constrained efficient allocation must satisfy the premium constraint (3) with $m_2 = 1$ as follows:

$$\begin{aligned} 0 = &\int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] \pi\left(\theta|\mathcal{I}_I\right) d\varphi d\theta \\ &+ \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] \pi\left(\theta|\mathcal{I}_I\right) d\varphi d\theta \\ &+ \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] \pi\left(\theta|\mathcal{I}_I\right) d\varphi d\theta. \end{aligned} \tag{C.1}$$

Next, consider a change to $\mathcal{I}_I = \mathcal{I}_\emptyset$ and $\mathcal{I}_S = \mathcal{I}_\theta$ without altering the planner's choice variables. Suppose that the premium ρ is too low to satisfy the constraint (3) after this change in information structure. From the external lenders' FOC for the bond position, such an outcome occurs if there is an inequality $>$ instead of an equality $=$ in that constraint.

On the one hand, if the country starts with $\beta = 0$, the information sensitivity shock moves it to subcase (a) (iii), so the second and third terms of constraint (3) are zero:

$$0 > \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta \text{ with } \beta = 0. \tag{C.2}$$

On the other hand, if the country starts with $\beta > 0$, the shock moves it to case (b), so the $m_2^{(b)}$ function applies:

$$\begin{aligned} 0 > &\int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta \\ &+ \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} m_2^{(b)} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta \\ &+ \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} m_2^{(b)} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta. \end{aligned} \tag{C.3}$$

Subtracting condition (C.1) from the relevant condition, (C.2) or (C.3), yields the expression in the lemma. ■

Proof of Proposition 3

Section B derives the FOCs of the system of equations.

Next, we use a perturbation approach to establish that there exist $m_2^{(b)}$ functions such that the planner selects a different value of $\frac{\beta}{\alpha}$ for case (b) relative to subcase (a) (i). Consider the following perturbation for case (b):

$$m_2^{(b)}\left(B_1, \eta_2, \{\eta_2\}_{\mathcal{E}_2 \in \widetilde{\Psi}}, \frac{\beta}{\alpha}\right) = 1 + \epsilon \widetilde{m}_2 \left(B_1, \eta_2, \{\eta_2\}_{\mathcal{E}_2 \in \widetilde{\Psi}}, \frac{\beta}{\alpha}\right).$$

Inserting this perturbation into the system of equations (2)-(4) and (5)-(8) and reorganizing them, we obtain a reduced system in the six variables $\{C_{F1}, \alpha, \beta, \gamma, \chi, \Lambda\}$. The first equation of the system is the rewritten premium constraint (3):

$$0 = \int_{\theta=0}^{1} \int_{\varphi=0}^{1} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta + z_p$$

$$\text{where } z_p = \epsilon \left[\begin{array}{c} \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} \widetilde{m}_2 \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta \\ + \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} \widetilde{m}_2 \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta \end{array}\right].$$

The next equations are the five FOCs (6)-(8) with the following LC and IS externality terms:

$$z_\chi = \epsilon \left\{\begin{array}{c} \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} \left\{\widetilde{m}_2 \frac{1}{\mathcal{E}_2} + \frac{\partial \widetilde{m}_2}{\partial \chi}\left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right]\right\} d\varphi d\theta \\ + \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} \left\{\widetilde{m}_2 \frac{1}{\mathcal{E}_2} + \frac{\partial \widetilde{m}_2}{\partial \chi}\left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right]\right\} d\varphi d\theta \end{array}\right.$$

$$z_{F1} = -\epsilon \left[\begin{array}{c} \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} \frac{\partial \widetilde{m}_2}{\partial B_1}\left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta \\ + \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} \frac{\partial \widetilde{m}_2}{\partial B_1}\left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta \end{array}\right]$$

$$z_{\omega 1} = \epsilon \left\{\begin{array}{c} \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} \left\{\widetilde{m}_2 \frac{\partial \mathcal{E}_2}{\partial \omega} \frac{\chi}{(\mathcal{E}_2)^2} - \frac{\partial \widetilde{m}_2}{\partial \omega}\left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right]\right\} d\varphi d\theta \\ + \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} \left\{\widetilde{m}_2 \frac{\partial \mathcal{E}_2}{\partial \omega} \frac{\chi}{(\mathcal{E}_2)^2} - \frac{\partial \widetilde{m}_2}{\partial \omega}\left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right]\right\} d\varphi d\theta \end{array}\right.$$

$$z_{\omega 2} = \frac{\epsilon}{2} \frac{\partial\left(-\frac{\beta}{\alpha}\right)}{\partial \omega} \left\{\begin{array}{c} \left[\frac{\chi}{\beta+\gamma} - (1+i^*)\right] \lim\limits_{\varphi \to \left(\frac{\beta}{\alpha}(1-\theta)\right)^-} \widetilde{m}_2 \\ + \left[\frac{\chi}{\alpha+\gamma} - (1+i^*)\right] \lim\limits_{\varphi \to \left(1-\frac{\beta}{\alpha}\theta\right)^+} \widetilde{m}_2 \end{array}\right\},$$

where we use the resource constraint (2), exchange rate equation (4), and AD wedge condition (5) to the substitute for the implicitly dropped variables $\{C_{F2}, \mathcal{E}_1, \mathcal{E}_2\}$. If preferred, the above system of equations can be adapted for $\mathcal{E}_2 = \widetilde{\alpha}\left[\varphi + \widetilde{\beta}\theta + \widetilde{\gamma}\right]$ (where $\widetilde{\alpha} = \alpha$, $\widetilde{\beta} = \frac{\beta}{\alpha}$, and $\widetilde{\gamma} = \frac{\gamma}{\alpha}$) by replacing $\omega \in \{\alpha, \beta, \gamma\}$ with $\omega \in \left\{\widetilde{\alpha}, \widetilde{\beta}, \widetilde{\gamma}\right\}$.

This reduced system of equations indicates that for the function $m_2^{(b)} = 1$ and $\widetilde{m}_2 = 0$, the planner selects the same value of all the variables including the ratio $\frac{\beta}{\alpha}$ in case (b) as they do in subcase (a) (i), while for the perturbation $m_2^{(b)} \neq 1$ and $\widetilde{m}_2 \neq 0$, all the variables including the ratio $\frac{\beta}{\alpha}$ are modified, except in knife-edge cases. ■

Proof of Corollary 1

We draw on the perturbation approach from the proof of Proposition 3 to establish that there exist $m_2^{(b)}$ functions such that the planner selects a lower value of $\frac{\beta}{\alpha}$ for case (b) relative to subcase (a) (i). In that proof, the perturbation $\widetilde{m}_2$ may generate an increase or a decrease in the ratio $\frac{\beta}{\alpha}$, except in knife-edge cases. If the perturbation causes a decrease in $\frac{\beta}{\alpha}$, the corollary is already proven. If the perturbation causes an increase in $\frac{\beta}{\alpha}$, it can be reversed to $-\widetilde{m}_2$ and there will then be a decrease in $\frac{\beta}{\alpha}$.

We prove the role of commitment by inspection: all the LC and IS externality terms in the FOCs are multiplied by Λ, which is positive when $B_1 > 0$. The constrained efficient allocation can be implemented directly by the planner or via delegation to a separate central banker, as long as the relevant authority can commit at $t = 1$ to the exchange rate function at $t = 2$, and also follows the same preferences and information structure as in our planner problem. ■

Proof of Proposition 4

The foreigners' FOC with respect to C^*_{F2} is as follows:

$$\frac{1}{C^*_{F2}} = \Upsilon,$$

where $\Upsilon\pi(\mathcal{E}_2|\mathcal{I}_L)$ is the multiplier on their budget constraint for exchange rate $\mathcal{E}_2 \in \Psi$. The external lenders' SDF is equal to the marginal value of a dollar of their profits Π_2 at each exchange rate $\mathcal{E}_2$. Accordingly, we obtain:

$$M_2 = \kappa\Upsilon = \frac{\kappa}{\kappa\Pi_2 + Y^*_{F2}}. \tag{C.4}$$

A country in case (b) has the insurable set $\widetilde{\Psi} \subset \Psi$ as indicated in Proposition 2. Within that set, Proposition 1 indicates that $(1+i^*)\,p(\mathcal{E}_2) = \pi(\mathcal{E}_2|\mathcal{I}_I)$. Since $\pi(\mathcal{E}_2|\mathcal{I}_L) = \pi(\mathcal{E}_2|\mathcal{I}_I)$, we obtain:

$$\overline{M} = \frac{\kappa}{\kappa\Pi_2 + Y^*_{F2}} \text{ for all } \mathcal{E}_2 \in \widetilde{\Psi}, \tag{C.5}$$

which means that $Q_L(\mathcal{E}_2)$ must be set such that the following condition holds:

$$\Pi_2 = \left[\eta_2 - (1+i^*)\right] B_1 + Q_L(\mathcal{E}_2) = \overline{\Pi} \text{ for all } \mathcal{E}_2 \in \widetilde{\Psi}. \tag{C.6}$$

Integrating this condition over $\mathcal{E}_2 \in \widetilde{\Psi}$ using the insurers' information structure $\mathcal{I}_I$, we obtain:

$$\overline{\Pi} = \frac{1}{1-\frac{\beta}{\alpha}} \left[\begin{array}{c} B_1 \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\eta_2 - (1+i^*)\right] d\varphi d\theta \\ + \int_{\mathcal{E}_2 \in \widetilde{\Psi}} Q_L(\mathcal{E}_2)\, \pi(\mathcal{E}_2|\mathcal{I}_I)\, d\mathcal{E}_2 \end{array} \right], \tag{C.7}$$

where the second term in the square brackets on the right hand side can be shown to be zero by inserting $p(\mathcal{E}_2) = \frac{\pi(\mathcal{E}_2|\mathcal{I}_I)}{(1+i^*)}$ into the lenders' insurance market constraint in subsection 2.1. Combining equations (C.4)-(C.7) yields the expression (10) in the proposition.

We substitute expression (10) into the premium constraint (3) to obtain:

$$\begin{aligned} 0 = {} & \frac{\int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta}{\frac{\kappa}{1-\frac{\beta}{\alpha}} B_1 \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] d\varphi d\theta + Y^*_{F2}} \\ & + \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} \frac{\left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right]}{\kappa B_1 \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] + Y^*_{F2}} d\varphi d\theta \\ & + \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} \frac{\left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right]}{\kappa B_1 \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*)\right] + Y^*_{F2}} d\varphi d\theta. \end{aligned}$$

Fixing the parameters $\{\alpha, \beta, \gamma\}$ in the above condition and taking derivatives with respect to χ and B_1 yields:

$$\frac{d\chi}{dB_1} = \frac{\kappa}{Y^*_{F2}} \frac{Z_1}{Z_2},$$

$$\text{where } Z_1 = \frac{\frac{1}{1-\frac{\beta}{\alpha}} \left[\int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*) \right] d\varphi d\theta \right]^2}{\left[\frac{\kappa}{1-\frac{\beta}{\alpha}} B_1 \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*) \right] d\varphi d\theta + Y^*_{F2} \right]^2}$$
$$+ \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} \frac{\left[\frac{\chi}{\mathcal{E}_2} - (1+i^*) \right]^2}{\left[\kappa B_1 \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*) \right] + Y^*_{F2} \right]^2} d\varphi d\theta$$
$$+ \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} \frac{\left[\frac{\chi}{\mathcal{E}_2} - (1+i^*) \right]^2}{\left[\kappa B_1 \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*) \right] + Y^*_{F2} \right]^2} d\varphi d\theta$$

$$Z_2 = \frac{\int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \frac{1}{\mathcal{E}_2} d\varphi d\theta}{\left[\frac{\kappa}{1-\frac{\beta}{\alpha}} B_1 \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*) \right] d\varphi d\theta + Y^*_{F2} \right]^2}$$
$$+ \int_{\theta=0}^{1} \int_{\varphi=0}^{\frac{\beta}{\alpha}(1-\theta)} \frac{\frac{1}{\mathcal{E}_2}}{\left[\kappa B_1 \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*) \right] + Y^*_{F2} \right]^2} d\varphi d\theta$$
$$+ \int_{\theta=0}^{1} \int_{\varphi=1-\frac{\beta}{\alpha}\theta}^{1} \frac{\frac{1}{\mathcal{E}_2}}{\left[\kappa B_1 \left[\frac{\chi}{\mathcal{E}_2} - (1+i^*) \right] + Y^*_{F2} \right]^2} d\varphi d\theta.$$

Since $\{Z_1, Z_2\} > 0$, this result establishes that $\frac{d\chi}{dB_1} > 0$. ■

Proof of Corollary 2

If inframarginal external lenders hold the dollar value S_1 of local currency bonds, the marginal lenders now have to hold $(B_1 - S_1)$ of such bonds. Correspondingly, their profit is $\Pi_2 = [\eta_2 - (1+i^*)] (B_1 - S_1) + Q_L(\mathcal{E}_2)$. By inspection of Proposition 4, the results established there continue to hold but with B_1 replaced by $B_1 - S_1$. ■

Proof of Proposition 5

First, we establish the m_2 function. The foreigners' FOC with respect to C^*_{F2} is as follows:

$$C_{F2}^{-\nu} = \Upsilon,$$

where $\Upsilon\pi(\mathcal{E}_2|\mathcal{I}_L)$ is the multiplier on their budget constraint for exchange rate $\mathcal{E}_2 \in \Psi$. The external lenders' SDF is equal to the marginal value of a dollar of their profits Π_2 at each exchange rate $\mathcal{E}_2$. Accordingly, we obtain:

$$M_2 = \Upsilon \left[1 + \varsigma\Theta(\Pi_2)(\underline{\Pi} - \Pi_2)\right] = \frac{\left[1 + \varsigma\Theta(\Pi_2)(\underline{\Pi} - \Pi_2)\right]}{\left[\Pi_2 - \varsigma\Theta(\Pi_2)\frac{(\underline{\Pi}-\Pi_2)^2}{2} + Y_2^*\right]^{\nu}}. \tag{C.8}$$

A country in case (b) has the insurable set $\widetilde{\Psi} \subset \Psi$ as indicated in Proposition 2. Within that set, Proposition

1 indicates that $(1+i^*)\, p(\mathcal{E}_2) = \pi(\mathcal{E}_2|\mathcal{I}_I)$. Since $\pi(\mathcal{E}_2|\mathcal{I}_L) = \pi(\mathcal{E}_2|\mathcal{I}_I)$, we obtain:

$$\overline{M} = \frac{\left[1+\varsigma\Theta(\Pi_2)\left(\underline{\Pi}-\Pi_2\right)\right]}{\left[\Pi_2 - \varsigma\Theta(\Pi_2)\frac{(\underline{\Pi}-\Pi_2)^2}{2} + Y_2^*\right]^{\nu}} \text{ for all } \mathcal{E}_2 \in \widetilde{\Psi}, \tag{C.9}$$

which means that irrespective of the value of $\Theta(\Pi_2)$, $Q_L(\mathcal{E}_2)$ must be set such that the following condition holds:

$$\Pi_2 = \left[\eta_2 - (1+i^*)\right] B_1 + Q_L(\mathcal{E}_2) = \overline{\Pi} \text{ for all } \mathcal{E}_2 \in \widetilde{\Psi}. \tag{C.10}$$

Analogously to the proof of Proposition 4, we can integrate this condition over $\mathcal{E}_2 \in \widetilde{\Psi}$, insert $p(\mathcal{E}_2) = \frac{\pi(\mathcal{E}_2|\mathcal{I}_I)}{(1+i^*)}$ into the lenders' insurance market constraint in subsection 2.1, and combine equations (C.8)-(C.10) to obtain:

$$\overline{\Pi} = \frac{1}{1-\frac{\beta}{\alpha}} B_1 \int_{\theta=0}^{1} \int_{\varphi=\frac{\beta}{\alpha}(1-\theta)}^{1-\frac{\beta}{\alpha}\theta} \left[\eta_2 - (1+i^*)\right] d\varphi d\theta \tag{C.11}$$

$$m_2 = \frac{1+\varsigma\Theta(\Pi_2)\left(\underline{\Pi}-\Pi_2\right)}{1+\varsigma\Theta\left(\overline{\Pi}\right)\left(\underline{\Pi}-\overline{\Pi}\right)} \left(\frac{\overline{\Pi} - \varsigma\Theta\left(\overline{\Pi}\right)\frac{\left(\underline{\Pi}-\overline{\Pi}\right)^2}{2} + Y_2^*}{\Pi_2 - \varsigma\Theta(\Pi_2)\frac{(\underline{\Pi}-\Pi_2)^2}{2} + Y_2^*}\right)^{\nu}. \tag{C.12}$$

Next, we prove by contradiction that $\frac{\chi}{\alpha+\beta+\gamma} < (1+i^*)$ in the constrained efficient allocation for case (b). Suppose instead that $\frac{\chi}{\alpha+\beta+\gamma} \geq (1+i^*)$. Since the allocation features $\alpha > \beta > 0$, $\mathcal{E}_2 = \alpha+\beta+\gamma$ is the strictly highest value of the exchange rate. Accordingly, we obtain $\frac{\chi}{\mathcal{E}_2} > (1+i^*)$ for all $\mathcal{E}_2 \in \Psi$ except possibly at $\varphi = \theta = 1$. Given that $m_2 > 0$, the premium constraint (3) cannot be satisfied with equality, so there is a contradiction.

Finally, we consider the following two scenarios for the parameter ς.

Scenario 1. For $\varsigma = 0$, we assume that the constrained efficient allocation for case (b) features $\frac{\beta}{\alpha} \in (0,1)$ and $B_1 > 0$. The allocation does not depend on the value of $\underline{\Pi}$. Since $\frac{\chi}{\alpha+\beta+\gamma} < (1+i^*)$, we can then set $\underline{\Pi}$ such that:

$$\underline{\Pi} = \left[\frac{\chi}{\alpha+\beta+\gamma} - (1+i^*)\right] B_1 < 0. \tag{C.13}$$

Since $\mathcal{E}_2 = \alpha+\beta+\gamma$ is the strictly highest value of the exchange rate, we also obtain that $\underline{\Pi} < \overline{\Pi}$.

Scenario 2. We switch to $\varsigma > 0$. For the value of $\underline{\Pi}$ in equation (C.13), the constrained efficient allocation is the same as in scenario 1. The reason is that for that value of $\underline{\Pi}$, $\Pi_2 \geq \underline{\Pi}$ for all $\mathcal{E}_2 \in \Psi$ even if we restrict $Q_L(\mathcal{E}_2) = 0$ throughout, so $\Theta(\Pi_2) = 0$ for all $\mathcal{E}_2 \in \Psi$. Now consider a marginal reduction in $\underline{\Pi}$ below the value in equation (C.13). The planner problem is continuous in the value of $\underline{\Pi}$, so there are marginal changes in $\frac{\beta}{\alpha}$ and B_1 which preserve their range and sign respectively. Moreover, we end up with $\Theta(\Pi_2) = 1$ only for some $\mathcal{E}_2 \in (\alpha+\gamma, \alpha+\beta+\gamma]$, which implies that $\Theta\left(\overline{\Pi}\right) = 0$ is preserved.

The argument in this scenario applies for any $\nu > 0$ and $\varsigma > 0$. Consider constrained efficient allocations which feature $\frac{\beta}{\alpha} \in (0,1)$ and $B_1 > 0$ for smaller and smaller values of ν, and then consider a marginal reduction in $\underline{\Pi}$ below the value in equation (C.13). In doing so, we obtain the expression (12) in the proposition. ■

Proof of Proposition 6

Combining the households' budget constraints with the market clearing conditions and the amended expression for the lump sum rebates, we obtain the amended resource constraints:

$$B_0(1+i^*) = (Y_{F1} - G_{F1} - C_{F1}) + (Y_{F2} - G_{F2} - C_{F2})\frac{\mathcal{E}_2}{\chi} \text{ for all } (\varphi, \theta),$$

and we amend the definition of B_1 as follows:

$$B_1 = B_0\left(1+i^*\right) + \left(C_{F1} + G_{F1} - Y_{F1}\right) = \left(Y_{F2} - G_{F2} - C_{F2}\right)\frac{\mathcal{E}_2}{\chi}.$$

Setting $Y_{Ft} = \overline{Y}_F$ and $A_t = \overline{A}$, we establish the proposition. ■

Proof of Proposition 7

The treatment of the resource constraints and the definition of B_1 are the same as in the proof of Proposition 6. We also amend the planner's period-t utility function V as indicated to incorporate the welfare obtained from government spending. The planner now optimizes over such spending, and the new FOCs with respect to G_{F1} and G_{F2} are as follows:

$$\frac{\sigma_{G1}}{G_{F1}} = \int_{\theta=0}^{1}\int_{\varphi=0}^{1}\Phi\left(\varphi,\theta\right)\pi\left(\theta|\mathcal{I}_I\right)d\varphi d\theta + z_{F1}$$

$$\frac{\delta\sigma_{G2}}{G_{F2}} = \Phi\left(\varphi,\theta\right)\frac{\mathcal{E}_2}{\chi}.$$

Combining these FOCs with those with respect to C_{F1} and C_{F2}, we obtain condition (13) in the proposition. ■

Proof of Corollary 3

Relative to the proof of Proposition 7, we impose the restriction that G_{F2} does not vary with θ. Correspondingly, we amend the FOC with respect to G_{F2} as follows:

$$\int_{\theta=0}^{1}\frac{\delta\sigma_{G2}}{G_{F2}}\pi\left(\theta|\mathcal{I}_I\right)d\theta = \int_{\theta=0}^{1}\Phi\left(\varphi,\theta\right)\frac{\mathcal{E}_2}{\chi}\pi\left(\theta|\mathcal{I}_I\right)d\theta.$$

Combining this FOC with the one with respect to C_{F2}, we obtain the amended condition (14) in the corollary.

Next, we consider the constrained efficient allocation for subcase (a) (i) subject to the restriction that G_{F2} does not vary with θ. To prove that the allocation features $\beta = 0$, we first establish the solution to a relaxed version of the planner problem and then show that this solution can be achieved by the unrelaxed planner problem described in Corollary 3. Since the country is in subcase (a) (i), we set $\pi\left(\theta|\mathcal{I}_I\right) = 1$ for all θ and $m_2 = 1$ for all $\mathcal{E}_2 \in \Psi$.

Relaxed planner problem. We relax the planner problem by replacing the exchange rate function (4) as follows:

$$\mathcal{E}_2 = \alpha\varphi + f\left(\theta\right), \tag{C.14}$$

where $f\left(\theta\right)$ is a relation that can depend on θ as well as constant terms. The relaxed planner can select a relation of the form $f\left(\theta\right) = \beta\theta + \gamma$, in which case the solution to the relaxed problem can be achieved by the unrelaxed planner problem, but they can also select other relations. Replacing the exchange rate function means that we must amend the premium constraint (3). Since $m_2 = 1$ for all $\mathcal{E}_2 \in \Psi$, it reduces to the following:

$$0 = \int_{\theta=0}^{1}\int_{\varphi=0}^{1}\left[\frac{\chi}{\mathcal{E}_2} - \left(1+i^*\right)\right]d\varphi d\theta. \tag{C.15}$$

The relaxed planner problem maximizes the utility function (1) subject to the constraints (2), (C.15), and (C.14), with $\{\beta,\gamma\}$ being replaced by $\{f\left(\theta\right)\}$ in the planner's choice variables. At the constrained efficient allocation, equations (5)-(7) hold with the settings $\pi\left(\theta|\mathcal{I}_I\right) = 1$ and $z_\chi = z_{F1} = z_{\omega1} = z_{\omega2} = 0$. Equation (8) also holds with those settings, but only for $\omega = \alpha$. There is a continuum of new FOCs with respect to $f\left(\theta\right)$ as follows, one for each value of θ:

$$\int_{\varphi=0}^{1}\left\{\frac{1}{\mathcal{E}_2}\delta\sigma_H\tau_{H2} + B_1\frac{1}{\mathcal{E}_2}\Phi\left(\varphi,\theta\right)\right\}d\varphi = \Lambda\int_{\varphi=0}^{1}\frac{\chi}{\left(\mathcal{E}_2\right)^2}d\varphi \text{ for all } \theta,$$

which we combine with the FOC for C_{F2} to obtain:

$$\int_{\varphi=0}^{1}\left\{\frac{1}{\mathcal{E}_2}\delta\sigma_H\tau_{H2}+B_1\frac{\chi}{(\mathcal{E}_2)^2}\frac{\delta\sigma_F}{C_{F2}}\left[1+\frac{\sigma_H}{\sigma_F}\tau_{H2}\right]\right\}d\varphi=\Lambda\int_{\varphi=0}^{1}\frac{\chi}{(\mathcal{E}_2)^2}d\varphi \text{ for all } \theta. \tag{C.16}$$

We can write τ_{H2} and C_{F2} as follows:

$$\tau_{H2}=1-\frac{\mathcal{E}_2 C_{F2}}{\sigma_F\overline{A}} \text{ and } C_{F2}=\overline{Y}_F-G_{F2}-B_1\frac{\chi}{\mathcal{E}_2},$$

so condition (C.16) is identical for each θ. As a result, for any given choice of α, the same value of $f(\theta)$ is selected for all θ. We can rewrite this solution as $f(\theta)=\beta\theta+\gamma$, where $\beta=0$.

Unrelaxed planner problem. Since the relaxed planner selects a relation of the form $f(\theta)=\beta\theta+\gamma$, the solution to the relaxed problem can be achieved by the unrelaxed planner problem described in Corollary 3. Correspondingly, the solutions of the two problems coincide. At the constrained efficient allocation for subcase (a) (i) subject to the restriction that G_{F2} does not vary with θ, we obtain $\beta=0$. Since C_{F2} and $\mathcal{E}_2$ both do not vary with θ, we can rewrite the condition for G_{F2} from equation (14) as follows:

$$G_{F2}=\frac{\int_{\theta=0}^{1}\sigma_{G2}d\theta}{\sigma_F}\frac{C_{F2}}{1+\frac{\sigma_H}{\sigma_F}\tau_{H2}}.$$

Finally, we consider the impact of the information sensitivity shock in Lemma 2 on the premium constraint (3). Since $\pi(\theta|\mathcal{I}_I)=1$ and $\beta=0$ prior to the change in the information structure, condition (9) is not met. Correspondingly, evaluated at the constrained efficient allocation for subcase (a) (i), the premium constraint remains satisfied after the change in the information structure. ■

Proof of Proposition 8

Combining the households' budget constraints with the market clearing conditions and the amended expression for the lump sum rebates, we obtain the amended resource constraints:

$$B_0(1+i^*)=(Y_{F1}-C_{F1})+\left(\widetilde{Y}_{F2}(\varphi)+\lambda(\theta,X)-X(1+i^*)-C_{F2}\right)\frac{\mathcal{E}_2}{\chi} \text{ for all } (\varphi,\theta),$$

and we amend the definition of B_1 as follows:

$$B_1=B_0(1+i^*)+(C_{F1}-Y_{F1})=\left(\widetilde{Y}_{F2}(\varphi)+\lambda(\theta,X)-X(1+i^*)-C_{F2}\right)\frac{\mathcal{E}_2}{\chi}.$$

These results establish the form of the planner problem. The planner's new FOC with respect to X is as follows:

$$\int_{\theta=0}^{1}\int_{\varphi=0}^{1}\left[\lambda_X(\theta,X)-(1+i^*)\right]\Phi(\varphi,\theta)\frac{\mathcal{E}_2}{\chi}\pi(\theta|\mathcal{I}_I)\,d\varphi d\theta=0.$$

Combining this FOC with the one with respect to C_{F2}, and then applying the covariance definition, we obtain condition (16) in the proposition. ■

Proof of Proposition 9

The planner commits to the institutional design with the highest welfare from the perspective of $t=0$.

- If the planner selects $\Delta=N$, the country is in case (b) at $t=1$, and the welfare from the perspective of $t=0$ is the same as the welfare from the perspective of $t=1$.

- If the planner selects $\Delta = S$, the country is in subcase (a) (i) at $t = 1$. The planner needs to subtract the cost of secrecy, yielding the expression for V^S in the proposition.
- If the planner selects $\Delta = T$, the country is in subcase (a) (ii) at $t = 1$, when all agents are informed about the value of θ. From the perspective of $t = 0$, the planner does not know the value of θ, so they must evaluate the welfare by integrating across all possible values of that private shock. They also need to account for the selection of $\{\alpha, \beta, \gamma\}$ at $t = 0$ and subtract the cost of collecting the information. These considerations yield the expression for V^T in the proposition. ■

PUBLICATIONS

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