Intro to Heterogeneity

Macro II - Fluctuations - ENSAE, 2024-2025

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Heterogeneity in models

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Example:

- *Macroeconomic Policy in DSGE and Agent-Based Models* from Revue de l'OFCE
- *In that respect, the Great Recessions has revealed to be a natural experiment for economic analysis, showing the inadequacy of the predominant theoretical frameworks. Indeed, an increasing number of leading economists claim that the current ''economic crisis is a crisis for economic theory'' (Kirman, 2010; Colander et al., 2009; Krugman, 2009, 2011; Caballero, 2010; Stiglitz, 2011; Kay, 2011; Dosi, 2011; Delong, 2011). The basic assumptions of mainstream DSGE models, e.g. rational expectations, representative agents, perfect markets etc., prevent the understanding of basic phenomena underlying the current economic crisis*

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But:

- mainstream models typically incorporate many non classical

Representative Agent

Under the **Representative agent** assumption

- aggregate choices are made as the result of a single optimization problem
- there might restrictions on what is internalized by the agent

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For the latter one needs a theory of aggregation[1]

- ... which quickly breaks down (for instance when utility fonction are heterogenous)

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Example with the Neoclassical Model

Let's consider three versions of the neoclassical model

- fully decentralized (many firms, many consumers)
- representative agent
- planner problem

Note:

- for the neoclassical model, there is a theory of aggregation for the production sector (firms are Cobb-Douglas)
- two assumptions are needed to aggregate consumers: log-utility and no uncertainty

Heterogenous Agents

Some economists have recognized early the need to explicitly model heterogeneity.

- **1977: Bewley**
  - idiosyncratic stochastic endowment
  - consumption-savings model with borrowing constraints
  - leads to *ex-post heterogeneity* (constrained/unconstrained) hence different reactions
- **Huggett Economy** (1993)
  - additional *ex-ante heterogeneity* in *idiosyncratic* income process
- **Ayiagari Model** (1994)
  - savings are invested to accumulate aggregate capital
  - consumption-savings model with borrowing constraints
  - idiosyncratic productivity shocks (salary)
- **Krussell Smith Model** (1998)
  - Ayagari + aggregate shocks

Mean Field Games and Heterogenous Agents Models

2012 Ben Moll did a talk at IMA (UK)

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The Theme of my Talk

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Result: a new stream of heterogenous agents papers

- PDE Models in Macroeconomics (2014) with Achdou, Bueary, Lasry, Lions
- The Dynamics of Inequality (2016) with Gabaix, Lasry, Lions
- Monetary Policy According to HANK (2018) with Kaplan and Violante
  - that one was hugely successfull

HANK, HANK HANK, ...

- *Monetary Policy According to HANK* (2018), by Moll, Kaplan and Violante
  - HANK: Heterogenous Agents New Keynesian
  - study unequal consequences of monetary policies
  - a new baseline model for central banks

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[3] pseudo representative new-keynesian model

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  - HANK: Heterogenous Agents New Keynesian
  - study unequal consequences of monetary policies
  - a new baseline model for central banks
- Stimulated a whole literature[2]
  - *Understanding HANK: Insights from a PRANK*[3]
  - *When HANK meets SAM*
  - *HANK beyond FIRE*
  - *Aggregate Demand: THANK (Tractable HANK) and TANK* by Florin Bilbiie
    - main point: you don't need more than two agents to get the main insights

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Heterogenous consumers

Why does it matter to model consumer's heterogeneity?

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Classically, we make the difference between two kinds of agents:

Ricardian Households

Agents who can freely re-allocate consumption intertemporally.
They have a high marginal propensity to consume out of additional income.

Keynesian Households

Agents whose consumption in the current period is limited by a binding credit constraint. Either they can't borrow at all or the amount they can borrow is limited today.
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Ricardian households choose not to consume more today, in order to consume more tomorrow.

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Let's have a look at the MPC distribution for France.[4]

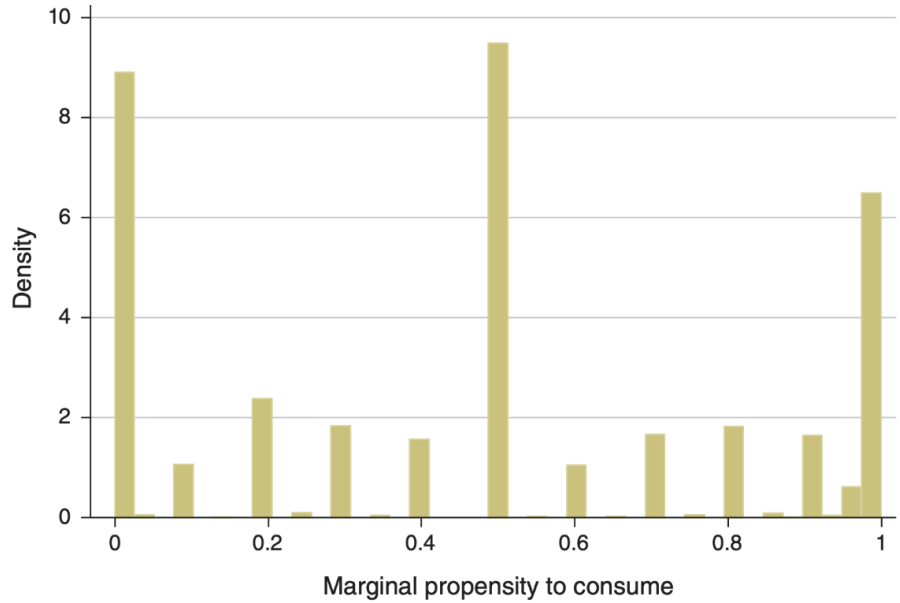


Figure 1: Self-Reported MPC from Transitory Income Shock

[4]From From Fiscal Policy and MPC Heterogeneity, Tullio Jappelli and Luigi Pistaferri, American Economic Journal: Macroeconomics, 2014

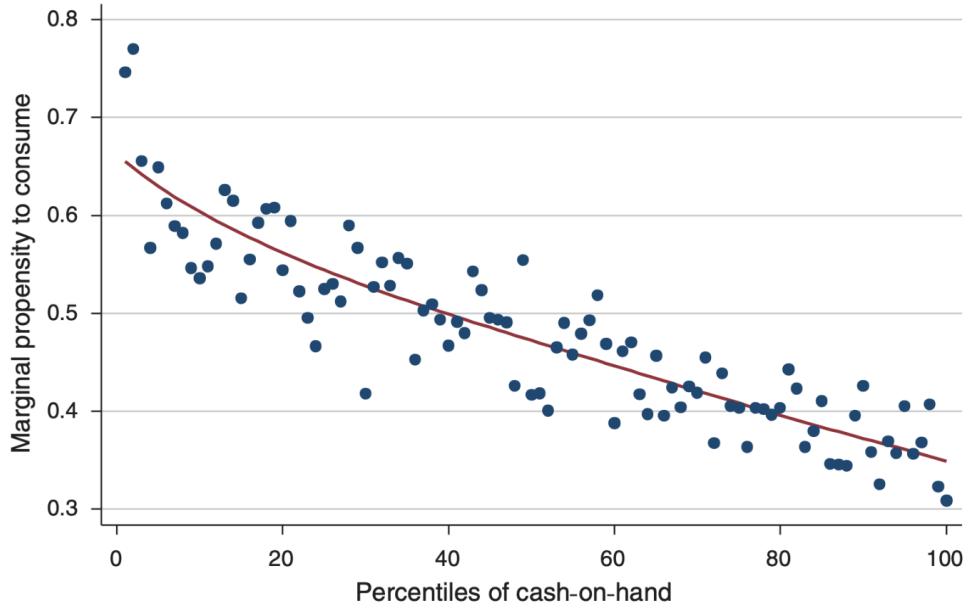
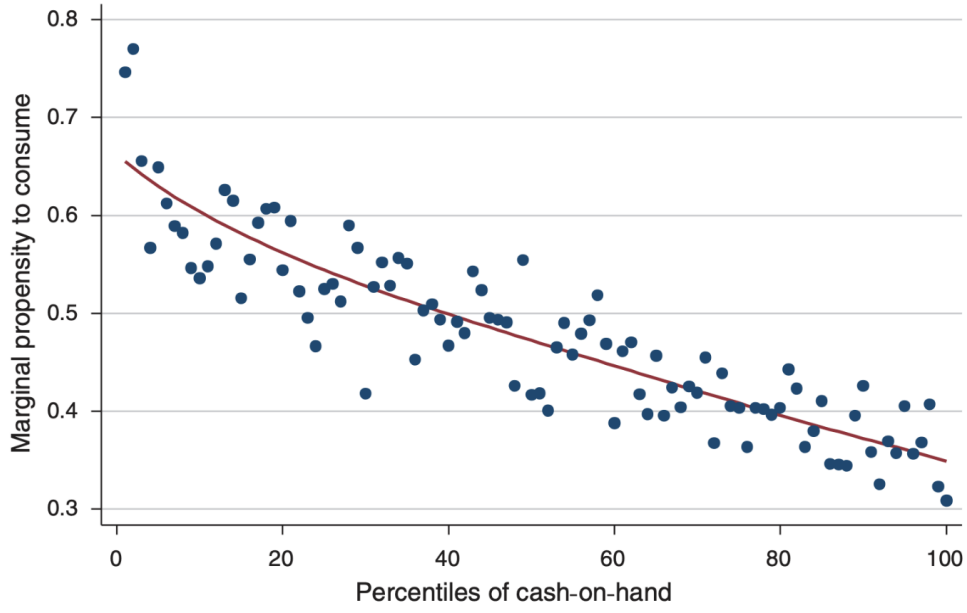


Figure 2: Average MPC by Cash-on-Hand Percentiles



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Apparently MPC is well predicted by Cash-in-hand (amount of money left to household after having made all compulsory payments).

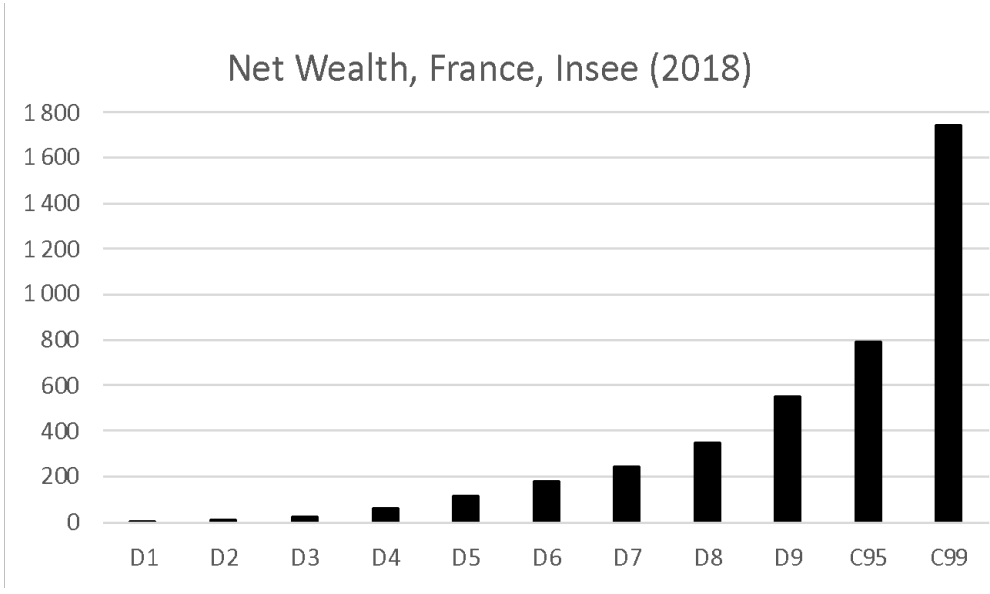


Figure 3: Wealth distribution

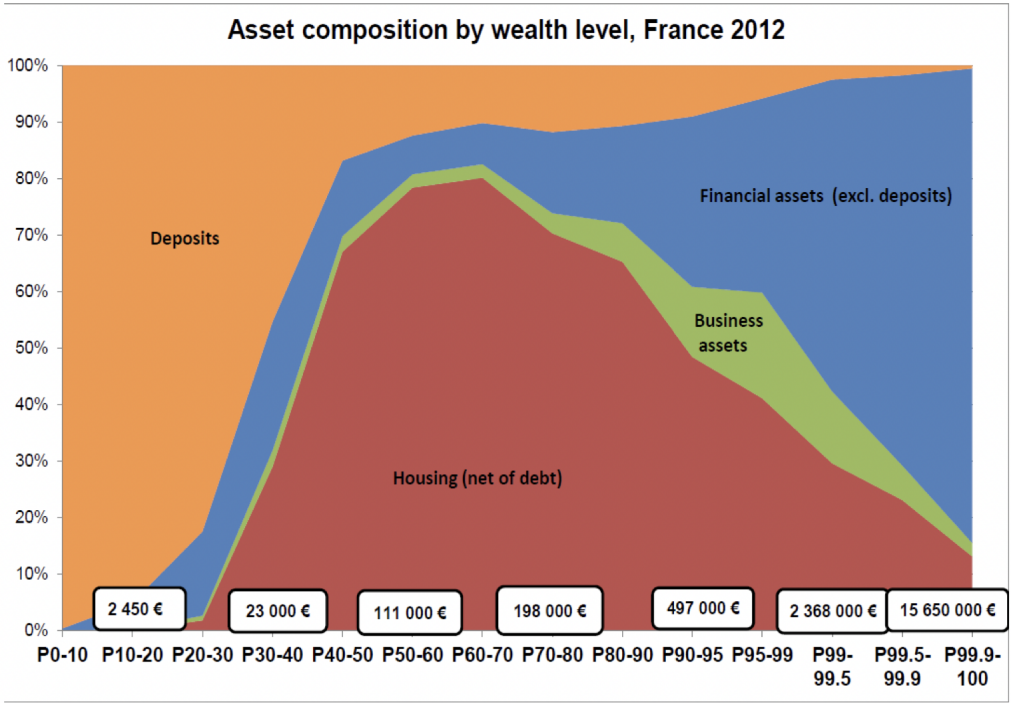
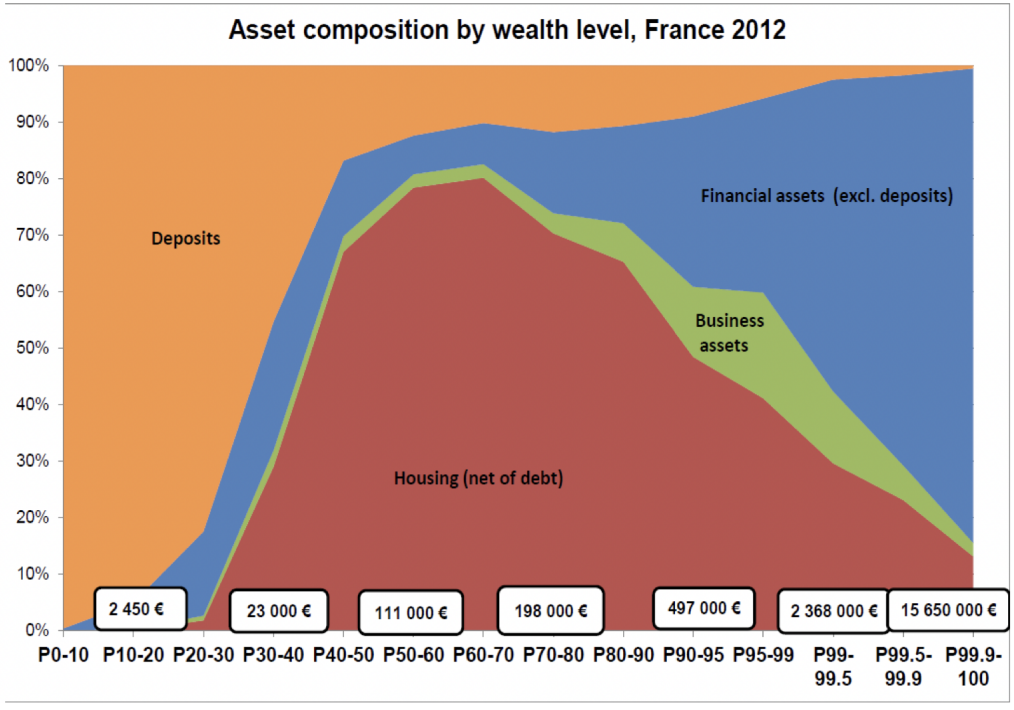


Figure 4: Wealth decomposition



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Agents in the middle of the wealth distribution have a mortgage, whose interests leaves very little to spend after payments. They have lower cash-in-hand hence higher marginal propensity to consume (than rich agents).

Wealthy Hand to Mouth agents

We have just seen that agents in the middle of the wealth distribution, hold a wider proportion of wealth in illiquid assets (housing)

- Their cash in hand (available for immediate purchase) is reduced. A sizable fraction of ther income goes into repaying their loan...).
- They have higher MPC
- They also react to interest rates changes (notably those who have floating interest rates)
- "Monetary Policy According to HANK", 2018, Kaplan, Moll and Violante, stress out the role of "wealthy hand to mouth" and the need to take their existence to evaluate the influence of monetary policies.

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- By using preference for wealth
  - coming next

Inequality, Leverage and Crisis

Inequality, Leverage and Crisis, Kumhof, Rancière, Winant (2015)

Introduction

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Ok, but from a macro perspective, what fueled such high levels of borrowing?

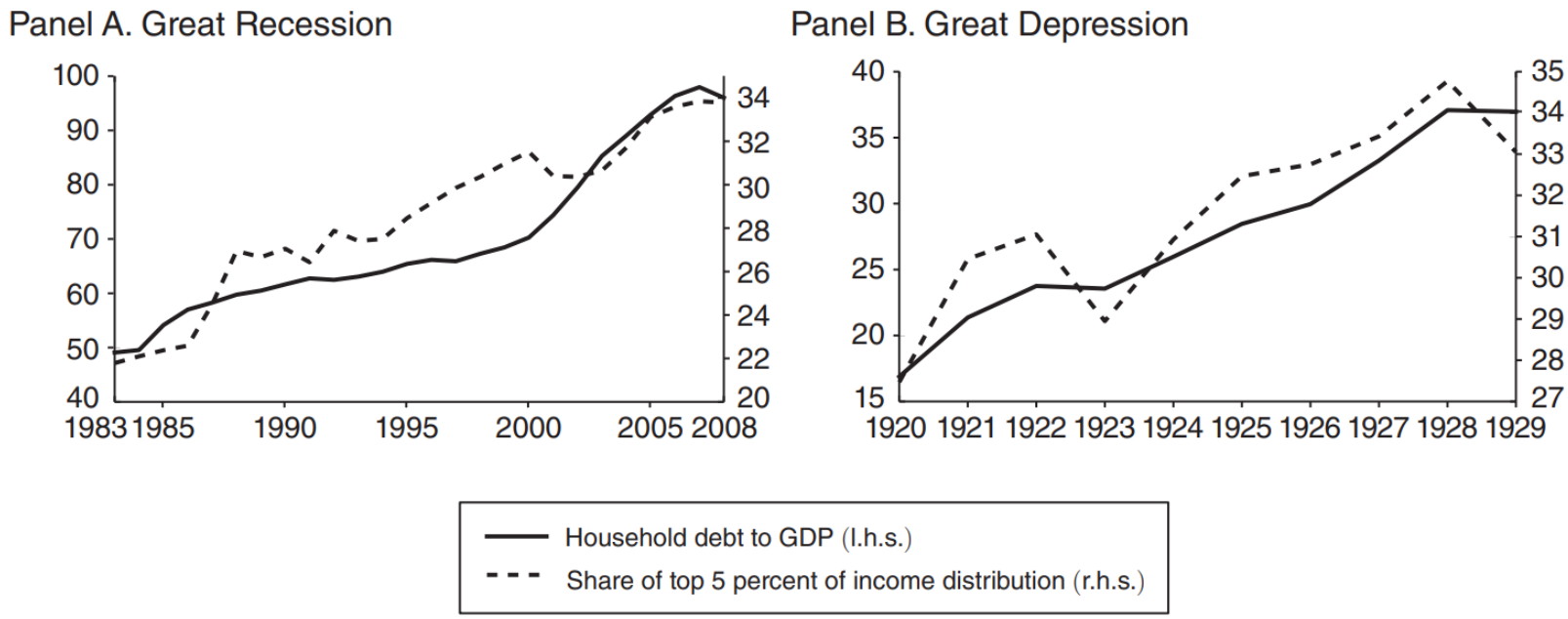

Figure 5: Leverage and Inequality

A similar pattern emerged before the great recession and before the great depression:[5]

- parallel increases in *income inequality* and debt over income ratios

[5] inequality data from Saez and Zucman

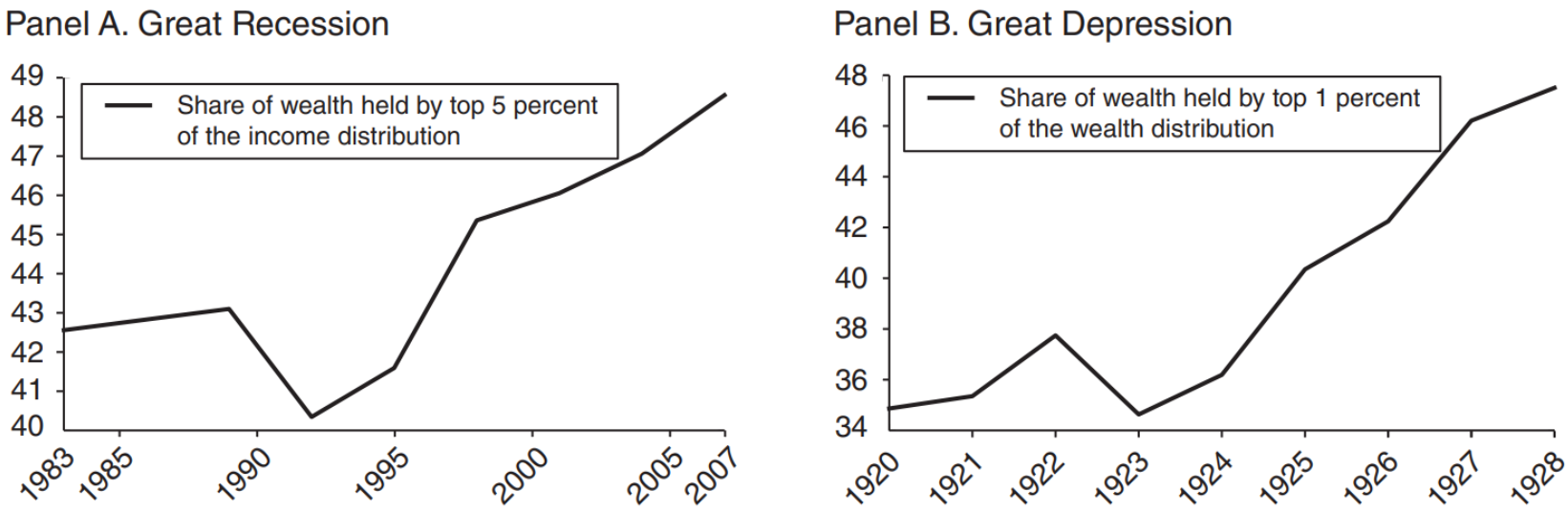

Figure 6: Wealth Inequality

Increase in *wealth inequality* is consistent.

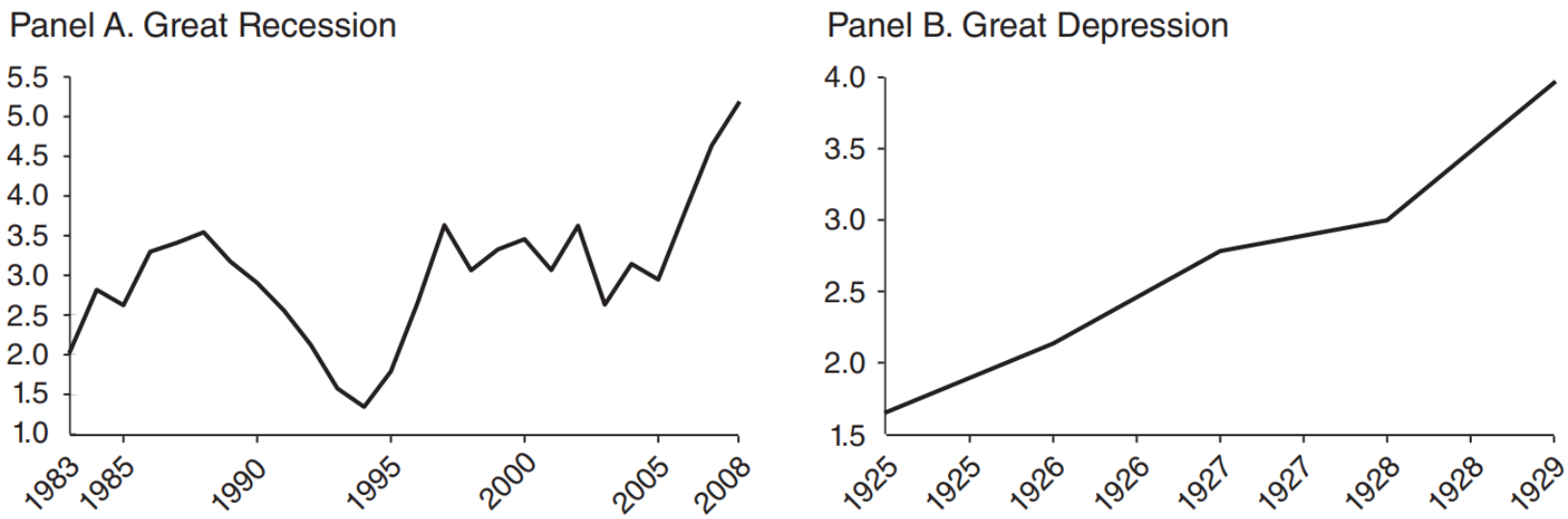

Figure 7: Crisis Probability

Econometric measures of household default risk [6] rose consistently.

[6] From Schularick and Taylor (2014)

Model

What could link rising income inequality to increased borrowing by bottom-earners?

Intuition:

- top-earners have higher marginal propensity to save
- when their income increases they lend to bottom earners
- and rising debt increases the risk of default

Let's see how to model that in DSGE fashion (ommiting default risk for the sake of simplicity)

Endowments

We consider and endowment economy:

- Total output

$$y_t = (1 - \rho_y)\overline{y} + \rho_y y_{t-1} + \epsilon_{y,t}$$

- Inequality shock

$$z_t = (1 - \rho_z)\overline{z} + \rho_z z_{t-1} + \epsilon_{z,t}$$

Comments:

- z_t is the fraction of the total output that is received by top-earners. The rest is received by bottom earners.
- We assume there is a faction χ of top earners.
- our goal is to study the effect of a persistent inequality shock (with $\rho_z = 1$)

Top Earners

We choose the following utility function for top earners:

$$U_t = E_t \sum_{k \geq 0}^{\infty} \beta_{\tau}^{k} \left\{ \frac{\left(c_{t+k}^{\tau}\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \varphi \frac{\left(1 + b_{t+k} \frac{1-\chi}{\chi}\right)^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}} \right\}$$

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Consumption:

$$c_t^\tau = y_t z_t \frac{1}{\chi} + \left(b_{t-1} - b_t p_t\right) \frac{1-\chi}{\chi}$$

where b_t is debt holdings and p_t the price of it $1/r_t$

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Optimality condition from $\max U_t$

$$p_t = \beta_\tau E_t \left[\left(\frac{c_{t+1}^\tau}{c_t^\tau} \right)^{-\frac{1}{\sigma}} \right] + \varphi \frac{\left(1 + b_t \frac{1-\chi}{\chi}\right)^{-\frac{1}{\eta}}}{\left(c_t^\tau\right)^{-\frac{1}{\sigma}}}$$

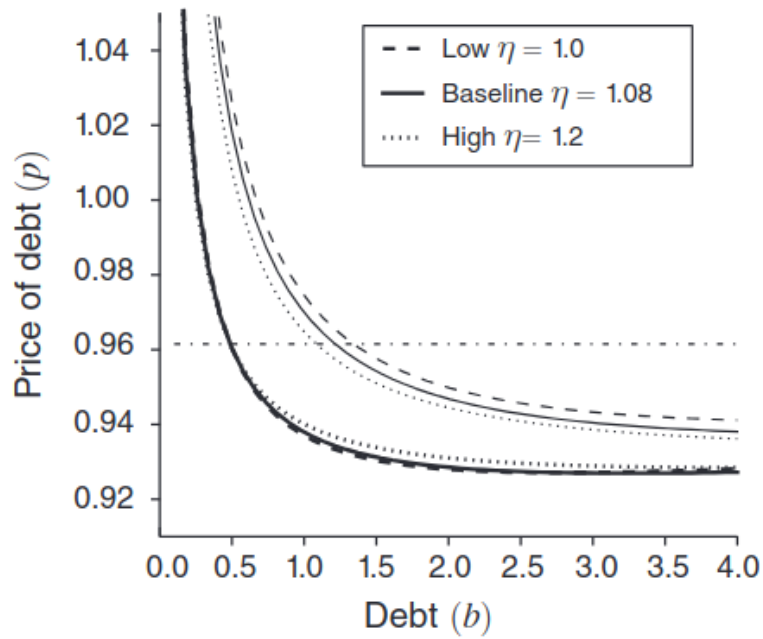
Preference for Wealth

The preference for wealth can be justified as:

- a preference for social status
- capitalist spirit

It implies a steady-state supply of lending for any income level:

$$\frac{\beta_b - \beta_\tau}{\varphi} = \frac{\left(\bar{y}\bar{z}\frac{1}{\chi} + \bar{b}(1-\beta_b)\frac{(1-\chi)}{\chi}\right)^{\frac{1}{\sigma}}}{\left(1 + \bar{b}\frac{(1-\chi)}{\chi}\right)^{\frac{1}{\eta}}}$$



Which in turn implies non-zero marginal propensity to save from a permanent income shock (in the short and the long run)

Parameters η and φ are not observed, but can be chosen in order

Bottom Earners

Bottom earners are standard:

$$V_t = E_t \sum_{k \geq 0}^{\infty} \beta_b^k \left(\frac{\left(c_{t+k}^b\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right)$$

Budget constraint:

$$c_t^b = y_t(1-z_t)\frac{1}{1-\chi} + (b_t p_t - b_{t-1})$$

Optimality condition from $\max V_t$

$$p_t = \beta^b E_t \left[\left(\frac{c_{t+1}^b}{c_t^b} \right)^{-\frac{1}{\sigma}} \right]$$

	Source/target	Implied values
Panel A. Directly calibrated parameters		
Steady-state output level	Normalization	$\overline{y} = 1$
Population share of top earners	5 percent	$\chi = 0.05$
Steady-state real interest rate	Literature	$\beta_b = 1.04^{-1}$
Ies in consumption	Literature	$\sigma = 0.5$
Panel B. Indirectly calibrated parameters		
Top earners' weight on wealth in utility	MPS of top earners	$\varphi = 0.05$
Top earners' wealth elasticity	MPS of top earners	$\eta = 1.09$
Steady-state top 5 percent income share $\overline{\tau}$	Data: 21.8 percent in 1983	$\overline{z} = 0.1807$
Steady-state debt-to-income ratio $\overline{\lambda}$	Data: 62.3 percent in 1983	$\beta_\tau = 0.912$
Panel C. Exogenous stochastic processes		
Output	Estimated	$\rho_y = 0.669$
		$\sigma_y = 0.012$
Output shares	Estimated	$\rho_z = 1$
		$\sigma_z = 0.008$

Figure 8: Calibration

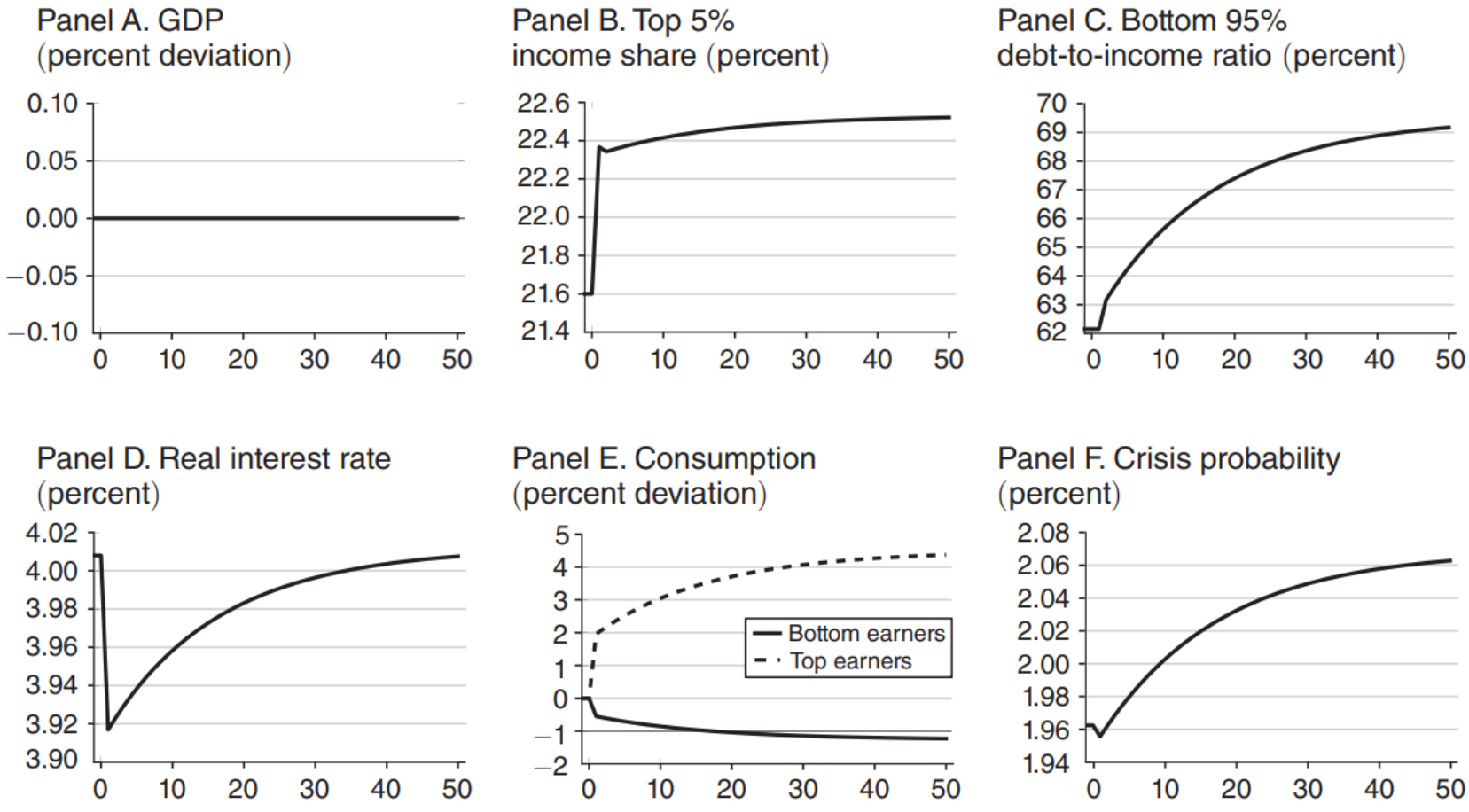


Figure 9: Inequality Shock

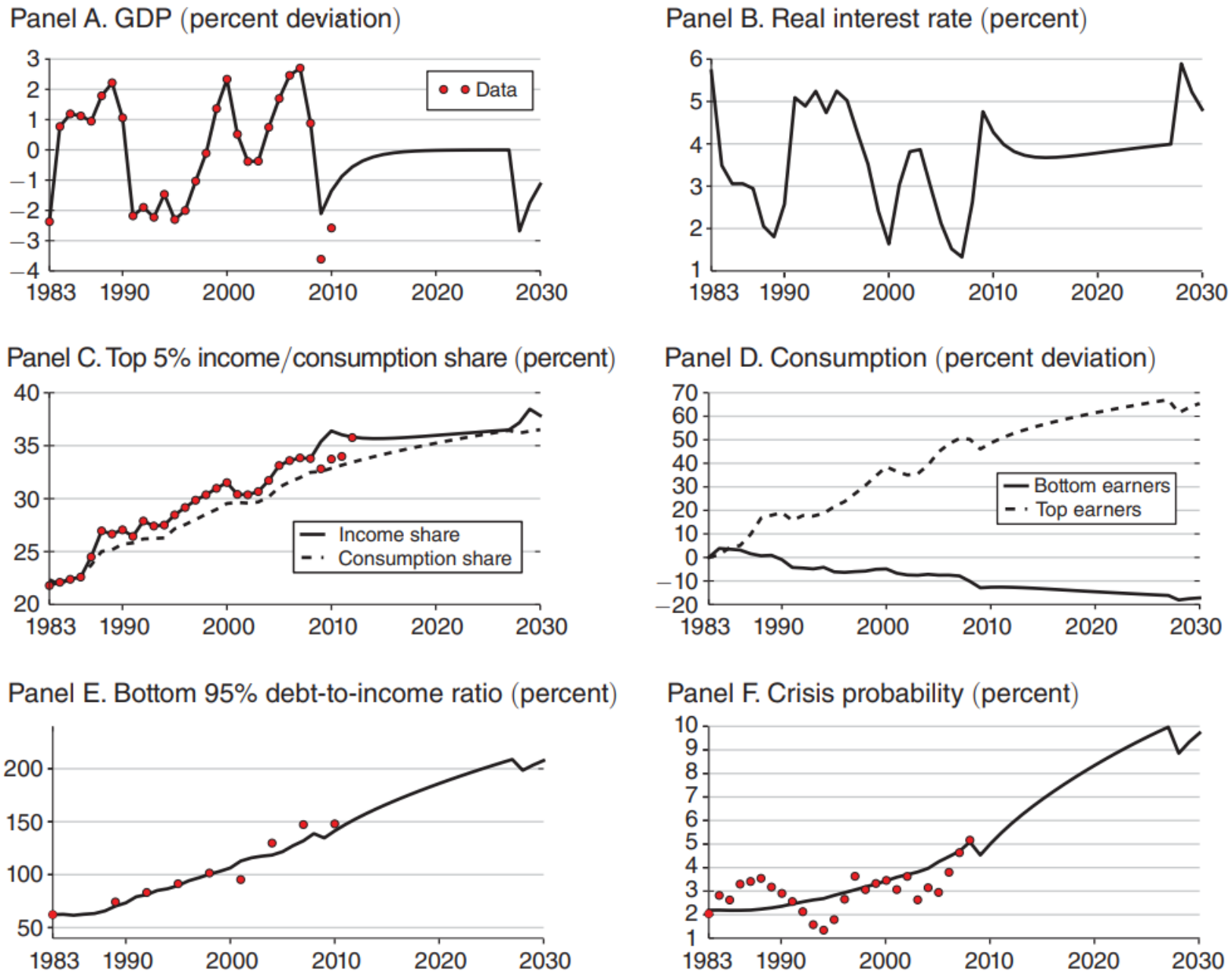


Figure 10: Pseudo-Historical Simulation

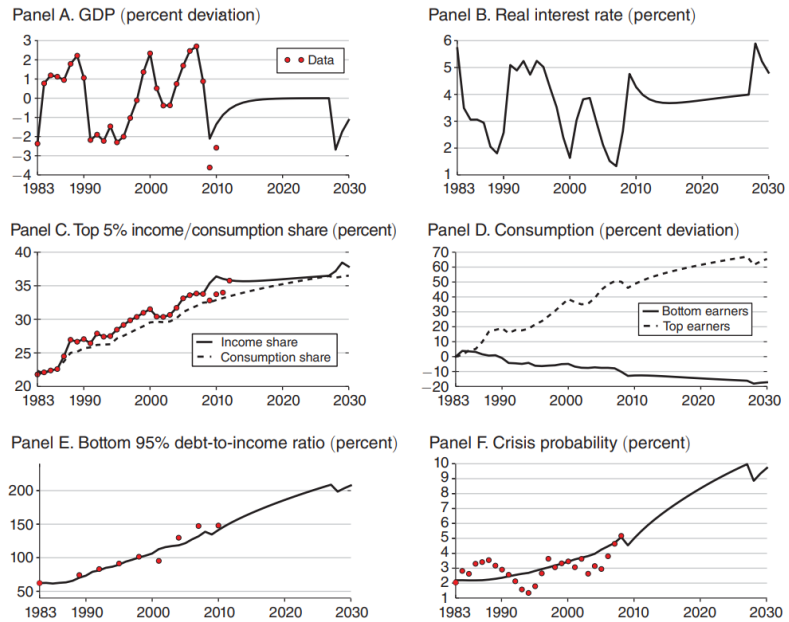


Figure 11: Pseudo-Historical Simulation

In the simulation we use historical values for the driving shocks (output and inequality). What is the predictive power of the model:

- we match *one* moment: the evolution of debt/gdep from 1983 to 2010