

Small Open Economy Extension (IRBC)

Macro II - Fluctuations - ENSAE, 2023-2024

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Introduction and Basic Facts

Why a small open economy?

What are the classical reasons to open economy to trade

- ▶ trade integration

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What are the classical reasons to open economy to trade

- ▶ trade integration
 - ▶ taste for variety
 - ▶ comparative advantage
- ▶ financial integration
 - ▶ smooth shock / insurance

From RBC to IRBC

After the success of RBC models to match business cycles it didn't take long before the same methodology was applied to International Business Cycles

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Seminal Paper:

- ▶ *International Real Business Cycles*, Backus, Kehoe, Kydland (1992) (freshwater economists)

Very successful methodology:

- ▶ facts at odd with theoretical predictions have been called “puzzles”

IRBC Facts

Properties of Business Cycles in OECD Economies

Country	Std. Dev. (%)		Ratio of Standard Deviation to that of y					Autocorr. y	Correlation with Output					
	y	nx	c	x	g	n	z	y	c	x	g	nx	n	z
Australia	1.45	1.23	.66	2.78	1.28	.34	1.00	.60	.46	.68	.15	-.01	.12	.98
Austria	1.28	1.15	1.14	2.92	.36	1.23	.84	.57	.65	.75	-.24	-.46	.58	.65
Canada	1.50	.78	.85	2.80	.77	.86	.74	.79	.83	.52	-.23	-.26	.69	.84
France	.90	.82	.99	2.96	.71	.55	.76	.78	.61	.79	.25	-.30	.77	.96
Germany	1.51	.79	.90	2.93	.81	.61	.83	.65	.66	.84	.26	-.11	.59	.93
Italy	1.69	1.33	.78	1.95	.42	.44	.92	.85	.82	.86	.01	-.68	.42	.96
Japan	1.35	.93	1.09	2.41	.79	.36	.88	.80	.80	.90	-.02	-.22	.60	.98
Switzerland	1.92	1.32	.74	2.30	.53	.71	.67	.90	.81	.82	.27	-.68	.84	.93
U.K.	1.61	1.19	1.15	2.29	.69	.68	.88	.63	.74	.59	.05	-.19	.47	.90
U.S.	1.92	.52	.75	3.27	.75	.61	.68	.86	.82	.94	.12	-.37	.88	.96
Europe	1.01	.50	.83	2.09	.47	.85	.98	.75	.81	.89	.10	-.25	.32	.85

Notes: Statistics are based on Hodrick-Prescott filtered data. Variables are: y, real output; c, real consumption; x, real fixed investment; g, real government purchases; nx, ratio of net exports to output, both at current prices; n, civilian employment; z, Solow residual, defined in text. Except for the ratio of net exports to output, statistics refer to logarithms of variables. Data are quarterly from the OECD's *Quarterly National Accounts*, except employment, which is from the OECD's *Main Economic Indicators*. The sample period is 1970:1 to 1990:2.

Figure 1: Moments

From Kehoe, Kydland (1995)

IRBC Facts

Table 1: IRBC Facts

Country	IRBC		IRBC		IRBC		IRBC		IRBC	
	1970-79	1980-89	1990-99	2000-09	2010-19	2020-29	2030-39	2040-49	2050-59	2060-69
Australia	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12
Austria	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12
Canada	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12
France	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12
Germany	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12
Italy	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12
Japan	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12
Switzerland	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12
United Kingdom	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12
EU	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12
EU	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12	1.18	1.12

International Comovements in OECD Economies

Country	Correlation with Same U.S. Variable					
	y	c	x	g	n	z
Australia	.51	-.19	.16	.23	-.18	.52
Austria	.38	.23	.46	.29	.47	.17
Canada	.76	.49	-.01	-.01	.53	.75
France	.41	.39	.22	-.20	.26	.39
Germany	.69	.49	.55	.28	.52	.65
Italy	.41	.02	.31	.09	-.01	.35
Japan	.60	.44	.56	.11	.32	.58
Switzerland	.42	.40	.38	.01	.36	.43
United Kingdom	.55	.42	.40	-.04	.69	.35
Europe	.66	.51	.53	.18	.33	.56

Notes: See Table 1.

Figure 3: Comoments

Figure 2:
Moments

Stylized Facts

Properties of House Cycles in OECD Countries

Country	N of Obs.	Ratio of Recession Duration to Expansion					Correlation with Output				
		1	2	3	4	5	1	2	3	4	5
Australia	124	1.25	1.14	1.04	0.94	0.84	0.81	0.79	0.77	0.75	0.73
Canada	124	1.14	1.04	0.94	0.84	0.74	0.71	0.69	0.67	0.65	0.63
France	124	1.04	0.94	0.84	0.74	0.64	0.61	0.59	0.57	0.55	0.53
Germany	124	0.94	0.84	0.74	0.64	0.54	0.51	0.49	0.47	0.45	0.43
Japan	124	0.84	0.74	0.64	0.54	0.44	0.41	0.39	0.37	0.35	0.33
UK	124	0.74	0.64	0.54	0.44	0.34	0.31	0.29	0.27	0.25	0.23
USA	124	0.64	0.54	0.44	0.34	0.24	0.21	0.19	0.17	0.15	0.13
OECD	124	0.54	0.44	0.34	0.24	0.14	0.11	0.09	0.07	0.05	0.03
EU	124	0.44	0.34	0.24	0.14	0.04	0.01	0.00	0.00	0.00	0.00
Japan	124	0.34	0.24	0.14	0.04	0.00	0.00	0.00	0.00	0.00	0.00

Note: Data are from the OECD Economic Outlook, Winter 2004. The sample period is 1970:1-2000:4. The correlation coefficients are based on the first differences of the log of the real GDP. The correlation coefficients are based on the first differences of the log of the real GDP. The correlation coefficients are based on the first differences of the log of the real GDP.

Domestically:

▶ output more variable than consumption

Figure 4:
Moments

International Comovements in OECD Economies

Country	Correlation with US GDP Growth				
	1	2	3	4	5
Australia	0.31	0.29	0.28	0.27	0.26
Canada	0.28	0.27	0.26	0.25	0.24
France	0.26	0.25	0.24	0.23	0.22
Germany	0.24	0.23	0.22	0.21	0.20
Japan	0.22	0.21	0.20	0.19	0.18
UK	0.20	0.19	0.18	0.17	0.16
OECD	0.18	0.17	0.16	0.15	0.14
EU	0.16	0.15	0.14	0.13	0.12

Note: See Table 1.

Figure 5:
Comovements

Internationally:

Stylized Facts

Properties of Output Growth in OECD Countries

Country	1970-1980					1980-1990				
	1	2	3	4	5	1	2	3	4	5
Australia	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Canada	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
France	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Germany	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Italy	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Japan	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
UK	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
USA	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Sweden	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Spain	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Switzerland	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Denmark	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Norway	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Finland	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Portugal	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Greece	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Spain	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12

Note: Data are from the OECD Growth Accounts. The data are quarterly real GDP growth rates, seasonally adjusted, in constant prices of 1995. The data are detrended and standardized to have a mean of zero and a standard deviation of one. The data are used to calculate the moments of the distribution of output growth rates. The moments are calculated as follows: 1. Mean: the average of the growth rates; 2. Variance: the average of the squared deviations from the mean; 3. Autocorrelation: the correlation between the growth rate in one period and the growth rate in the next period; 4. Skewness: the third moment of the distribution; 5. Kurtosis: the fourth moment of the distribution. The data are used to calculate the moments of the distribution of output growth rates. The moments are calculated as follows: 1. Mean: the average of the growth rates; 2. Variance: the average of the squared deviations from the mean; 3. Autocorrelation: the correlation between the growth rate in one period and the growth rate in the next period; 4. Skewness: the third moment of the distribution; 5. Kurtosis: the fourth moment of the distribution.

Figure 4:
Moments

Domestically:

- ▶ output more variable than consumption
- ▶ output autocorrelated
- ▶ productivity strongly procyclical

International Comovements in OECD Economies

Country	Correlations with US GDP Growth				
	1	2	3	4	5
Australia	0.11	-0.10	0.10	0.10	-0.10
Canada	0.10	0.10	0.10	0.10	0.10
France	0.10	0.10	0.10	0.10	0.10
Germany	0.10	0.10	0.10	0.10	0.10
Italy	0.10	0.10	0.10	0.10	0.10
Japan	0.10	0.10	0.10	0.10	0.10
UK	0.10	0.10	0.10	0.10	0.10
USA	0.10	0.10	0.10	0.10	0.10
Sweden	0.10	0.10	0.10	0.10	0.10
Spain	0.10	0.10	0.10	0.10	0.10
Switzerland	0.10	0.10	0.10	0.10	0.10
Denmark	0.10	0.10	0.10	0.10	0.10
Norway	0.10	0.10	0.10	0.10	0.10
Finland	0.10	0.10	0.10	0.10	0.10
Portugal	0.10	0.10	0.10	0.10	0.10
Greece	0.10	0.10	0.10	0.10	0.10
Spain	0.10	0.10	0.10	0.10	0.10

Note: See Table 1.

Figure 5:
Comovements

Internationally:

Stylized Facts

Properties of Output, Capital & Trade Moments

Country	N of Obs.	Output					Capital					Trade					
		1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	
Australia	124	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Canada	124	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
France	124	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Germany	124	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Japan	124	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
U.S.	124	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Europe	124	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12

Note: Moments are calculated for the period 1970:1-1995:4. The sample size varies by country due to data availability. The first column shows the number of observations. The following columns show the moments of the distribution of the variables. The first column shows the mean, the second column shows the standard deviation, the third column shows the skewness, the fourth column shows the kurtosis, and the fifth column shows the excess kurtosis. The last column shows the correlation of the variable with the U.S. variable.

Figure 4:
Moments

Domestically:

- ▶ output more variable than consumption
- ▶ output autocorrelated
- ▶ productivity strongly procyclical
- ▶ trade balance strongly countercyclical

International Comovements in OECD Economies

Country	Correlations with U.S. Variable				
	Y	K	T	X	Z
Australia	.33	-.29	.38	.20	-.18
Canada	.36	.33	.48	.47	.17
France	.38	.49	-.26	-.25	.73
Germany	.41	.59	.22	-.23	.38
Japan	.49	.49	.20	.24	.48
Spain	.45	.61	.31	.39	.33
Sweden	.45	.49	.29	.26	.44
United Kingdom	.31	.41	.48	-.36	.48
Europe	.48	.31	.33	.13	.31

Note: See Table 1.

Figure 5:
Comovements

Internationally:

Stylized Facts

Properties of Output Growth in OECD Countries

Country	1970-1980					1980-1990				
	1	2	3	4	5	1	2	3	4	5
Canada	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
France	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Germany	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Italy	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Japan	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
UK	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
USA	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Sweden	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Spain	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Denmark	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Average	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12

Note: Data are from the OECD Economic Outlook, 1991, Table 1.1. The data are quarterly real GDP growth rates, seasonally adjusted, in constant prices. The sample period is 1970:1-1989:4. The average is calculated over all countries in the sample.

Figure 4:
Moments

Domestically:

- ▶ output more variable than consumption
- ▶ output autocorrelated
- ▶ productivity strongly procyclical
- ▶ trade balance strongly countercyclical
- ▶ positive comovements in output

Internationally:

International Comovements in OECD Economies

Country	Correlation with US GDP Growth				
	1	2	3	4	5
Australia	.31	-.29	.28	.20	-.18
Belgium	.38	.33	.48	.47	.17
Canada	.78	.69	-.26	-.26	.73
Denmark	.41	.39	.22	-.23	.28
Germany	.69	.69	.20	.24	.32
Italy	.45	.61	.31	.28	-.22
Japan	.45	.61	.31	.28	-.22
France	.45	.61	.31	.28	-.22
UK	.45	.61	.31	.28	-.22
Spain	.45	.61	.31	.28	-.22
Average	.45	.61	.31	.28	-.22

Note: See Table 1.

Figure 5:
Comovements

Stylized Facts

Properties of Output Growth in OECD Countries

Country	OECD Average					Germany					Australia and Japan					
	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	
Output	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Consumption	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Productivity	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Trade Balance	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12

Note: See Table 1.

Figure 4:
Moments

Domestically:

- ▶ output more variable than consumption
- ▶ output autocorrelated
- ▶ productivity strongly procyclical
- ▶ trade balance strongly countercyclical
- ▶ positive comovements in output

Internationally:

- ▶ smaller comovements in consumption

International Comovements in OECD Economies

Country	Comovements with US GDP Growth				
	1	2	3	4	5
Australia	21	-23	28	22	-18
France	38	23	48	47	17
Germany	38	49	-26	-25	23
Japan	41	39	22	-23	28
United Kingdom	49	49	20	24	22
Spain	45	41	21	28	-22
Sweden	45	49	28	21	12
Denmark	45	49	28	21	12
United States	21	41	48	48	18
Europe	48	21	23	13	22

Note: See Table 1.

Figure 5:
Comovements

Stylized Facts

Properties of Output Growth in OECD Countries

Country	1970-1980					1980-1990				
	1	2	3	4	5	1	2	3	4	5
Canada	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
France	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Germany	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Italy	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Japan	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
UK	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
USA	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Sweden	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Spain	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
Average	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12

Notes: All figures are in percent. The first column shows the average of the five variables. The second column shows the average of the five variables. The third column shows the average of the five variables. The fourth column shows the average of the five variables. The fifth column shows the average of the five variables. The sixth column shows the average of the five variables. The seventh column shows the average of the five variables. The eighth column shows the average of the five variables. The ninth column shows the average of the five variables. The tenth column shows the average of the five variables. The eleventh column shows the average of the five variables. The twelfth column shows the average of the five variables. The thirteenth column shows the average of the five variables. The fourteenth column shows the average of the five variables. The fifteenth column shows the average of the five variables. The sixteenth column shows the average of the five variables. The seventeenth column shows the average of the five variables. The eighteenth column shows the average of the five variables. The nineteenth column shows the average of the five variables. The twentieth column shows the average of the five variables. The twenty-first column shows the average of the five variables. The twenty-second column shows the average of the five variables. The twenty-third column shows the average of the five variables. The twenty-fourth column shows the average of the five variables. The twenty-fifth column shows the average of the five variables. The twenty-sixth column shows the average of the five variables. The twenty-seventh column shows the average of the five variables. The twenty-eighth column shows the average of the five variables. The twenty-ninth column shows the average of the five variables. The thirtieth column shows the average of the five variables. The thirty-first column shows the average of the five variables. 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Figure 4: Moments

Domestically:

- ▶ output more variable than consumption
- ▶ output autocorrelated
- ▶ productivity strongly procyclical
- ▶ trade balance strongly countercyclical
- ▶ positive comovements in output

International Comovements in OECD Economies

Country	Correlation with US GDP Growth				
	1	2	3	4	5
Australia	0.1	0.1	0.1	0.1	0.1
Belgium	0.1	0.1	0.1	0.1	0.1
Canada	0.1	0.1	0.1	0.1	0.1
Denmark	0.1	0.1	0.1	0.1	0.1
France	0.1	0.1	0.1	0.1	0.1
Germany	0.1	0.1	0.1	0.1	0.1
Italy	0.1	0.1	0.1	0.1	0.1
Japan	0.1	0.1	0.1	0.1	0.1
Netherlands	0.1	0.1	0.1	0.1	0.1
Spain	0.1	0.1	0.1	0.1	0.1
Sweden	0.1	0.1	0.1	0.1	0.1
UK	0.1	0.1	0.1	0.1	0.1
USA	0.1	0.1	0.1	0.1	0.1
Average	0.1	0.1	0.1	0.1	0.1

Notes: See Table 1.

Figure 5: Comovements

Internationally:

- ▶ smaller comovements in consumption
- ▶ Backus-Kehoe-Kydland puzzle

Modeling a Small Open Economy

Endowment model

Take an endowment economy: income $(y_t)_t$ is exogenously given. We assume it is deterministic

$$\max_{c_t} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$c_t + a_{t+1} \leq y_t + (1+r)a_t$$

Country takes world interest rate r as given

- ▶ a small open economy doesn't affect world prices

Endowment model (2)

We solve this problem with the terminal conditions:

▶ a_0 given

▶ $\lim_{T \rightarrow \infty} \frac{a_{T+1}}{(1+r)^T} \geq 0$

▶ *no-ponzi* condition

Endowment model (2)

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▶ a_0 given

▶ $\lim_{T \rightarrow \infty} \frac{a_{T+1}}{(1+r)^T} \geq 0$

▶ *no-ponzi* condition

The no-ponzi condition will in effect eliminate diverging solutions. In a first order approximation, it selects the right eigenvalues.

Endowment model (3)

We get the lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c_t) + \sum_{t=0}^{\infty} \beta^t \lambda_t (y_t + (1+r)a_t - c_t - a_{t+1})$$

First order conditions:

$$u'(c_t) = \lambda_t \quad (1)$$

$$\lambda_t = \beta(1+r)\lambda_{t+1} \quad (2)$$

Under the *technical assumption* $\beta(1+r) = 1$ we get:

$$c_0 = \frac{r}{1+r} \left\{ (1+r)a_0 + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} \right\}$$

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► problem isomorphic to consumption-savings decisions

Current Account

Reminders on Current Account

The **trade balance** is exports-imports (here $y_t - c_t$)

The **current account** is trade balance + net factor payments
(here $y_t - c_t + ra_t$)

Positive **current account**: additional lending to the rest of the world.

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💡 Reminders on Current Account

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Positive **current account**: additional lending to the rest of the world.

Using the formula from before

$$CA_0 = a_0 r + \left(1 - \frac{r}{1+r}\right) y_0 - \frac{r}{1+r} \left\{ \sum_{t \geq 1}^{\infty} \frac{y_t}{(1+r)^t} \right\}$$

How does the current account reacts to income shocks?

- ▶ current account responds positively to *temporary* shock in income
- ▶ and to news about future income shocks:

Unit root

Still with the same formula:

$$c_0 = \frac{r}{1+r} \left\{ (1+r)a_0 + \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} \right\}$$

What is the effect of an increase in a_0 ?

- ▶ consumption rises permanently
 - ▶ by small amount r corresponding to interests paid forever on a_0
- ▶ this will correspond to a unit root in the solution

Exercise

From the first order conditions

$$u'(c_t) = \lambda_t \quad (3)$$

$$\lambda_t = \beta(1+r)\lambda_{t+1} \quad (4)$$

assuming $u(c_t) = \log(c_t)$, can you get the equation for the law of motion of a_t and show the presence of a unit root?

Adding capital

We add capital and production to our endowment economy:

$$y_t = z_t k_t^\alpha$$

$$k_t = (1 - \delta)k_{t-1} + i_{t-1}$$

The aggregate resource constraint becomes:

$$a_{t+1} + c_t + i_t = (1 + r)a_t + y_t$$

Now maximize $\sum_t \beta^t U(c_t)$

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Now maximize $\sum_t \beta^t U(c_t)$

We get first order conditions

$$\lambda_t = \beta \lambda_{t+1} (1 + r)$$

$$\lambda_t = \beta \lambda_{t+1} [(1 - \delta) + z_{t+1} f'(k_{t+1})]$$

where λ_t is lagrange multiplier associated to budget constraint.

Adding capital: optimality conditions

Since λ_t (constraint is always binding), we get:

$$(1 - \delta) + z_{t+1}f'(k_{t+1}) = 1 + r$$

$$k_{t+1} = \left(\frac{r + \delta}{\alpha z_{t+1}} \right)^{\frac{1}{\alpha-1}}$$

and investment

$$i_t = \left(\frac{r + \delta}{\alpha z_{t+1}} \right)^{\frac{1}{\alpha-1}} - (1 - \delta) \left(\frac{r + \delta}{\alpha z_t} \right)^{\frac{1}{\alpha-1}}$$

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Here investment is fully determined by productivity shocks

► too simple: no international dependence

Add friction to the investment

A possible solution: change the resource constraint such that adjusting capital is costly

For instance:

$$a_{t+1} + c_t + i_t + \frac{\omega (k_{t+1} - k_t)^2}{2 k_t} = (1 + r)a_t + z f(k_t)$$

$$k_{t+1} = (1 - \delta)k_t + i_t$$

where ω is an adjustment friction. Typically, ω is chosen so that the model replicates $\frac{Var(i_t)}{Var(y_t)}$ from the data.

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Cf tutorial.

A benchmark Small Open Economy Model

A benchmark Small Open Economy Model



Stephanie Schmitt-Grohe and Martin Uribe.

Figure 6: Stephanie Schmitt Grohe and Martin Uribe

Closing Small Economy Models,
Schmitt Grohe and Uribe
(2003), JIE

- ▶ small open economy model with production, consumption-leisure tradeoff and capital adjustment costs
 - ▶ = RBC+open+adj costs
- ▶ perform some moments matching
- ▶ compare different ways of stationarizing the model

The model

$$\max_{c_t, n_t} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$c_t + k_{t+1} + a_{t+1} = y_t + g_t - \frac{\omega}{2} (k_{t+1} - k_t)^2 + (1 - \delta)k_t + (1 + r^* + \pi(a_t))a_t$$

$$y_t = f(k_t, n_t, z_t)$$

$$z_{t+1} = \rho z_t + \epsilon_{t+1}$$

$$\text{and } u(c, n) = \frac{1}{1 - \sigma} (c^\psi (1 - n)^{1 - \psi})^{1 - \sigma}$$

How to make the distribution stationary?

The solution of the model exhibits a unit root:

$$a_t = a_{t-1} + \dots \text{other variables in } t-1 + \text{shocks in } t$$

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Problem:

- ▶ there isn't a unique deterministic steady-state
- ▶ the ergodic distribution of the model variables is not defined

This raises practical issues (notably for estimation) for the *linear* model.

- ▶ no unconditional moments

How to get rid of the unit root?

General idea:

- ▶ introduce a force that pulls the level of foreign assets towards equilibrium

Schmitt Grohe and Uribe (2003) consider many options:

- ▶ debt-elastic interest rate:

$$1 + r = 1 + r^* + \pi(a_d)$$

- ▶ with $\pi(0) = 0$ and $\pi'(0) > 0$
- ▶ π can be understood as a risk premium on rising debt
- ▶ endogenous time-discount (aka Usawa preferences)

$$\beta(c_t) = (1 + c_t)^{-\chi}$$

- ▶ costs of adjustment for international portfolios

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- ▶ costs of adjustment for international portfolios

SGU show that the choice of the stationarization device has little effect for the dynamics (moments) of most variables

Calibration

Parameters	Values
σ	2
ψ	1.45
α	0.32
ω	0.028
r	0.04

Parameters	Values
δ	0.1
ρ	0.42
σ^2	0.0129
A^*	-0.7442
χ	0.000742

Results

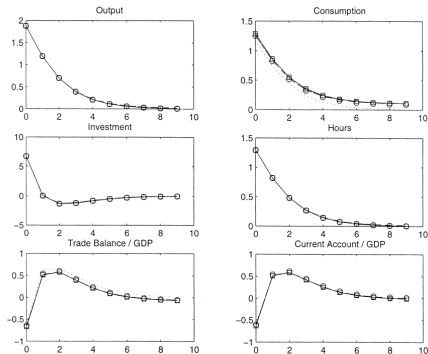


Fig. 1. Impulse response to a unit technology shock in Models 1–5. Note. Solid line: Endogenous discount factor model; Squares: Endogenous discount factor model without internalization; Dashed line: Debt-elastic interest rate model; Dash-dotted line: Portfolio adjustment cost model; Dotted line: complete asset markets model; Circles: Model without stationarity inducing elements.

Figure 7: Impulse Response Function

Table 3
Observed and implied second moments

	Data	Model 1	Model 1a	Model 2	Model 3	Model 4
<i>Volatilities:</i>						
$\text{std}(y_t)$	2.8	3.1	3.1	3.1	3.1	3.1
$\text{std}(c_t)$	2.5	2.3	2.3	2.7	2.7	1.9
$\text{std}(i_t)$	9.8	9.1	9.1	9	9	9.1
$\text{std}(h_t)$	2	2.1	2.1	2.1	2.1	2.1
$\text{std}\left(\frac{ib_t}{y_t}\right)$	1.9	1.5	1.5	1.8	1.8	1.6
$\text{std}\left(\frac{ca_t}{y_t}\right)$		1.5	1.5	1.5	1.5	
<i>Serial correlations:</i>						
$\text{corr}(y_t, y_{t-1})$	0.61	0.61	0.61	0.62	0.62	0.61
$\text{corr}(c_t, c_{t-1})$	0.7	0.7	0.7	0.78	0.78	0.61
$\text{corr}(i_t, i_{t-1})$	0.31	0.07	0.07	0.069	0.069	0.07
$\text{corr}(h_t, h_{t-1})$	0.54	0.61	0.61	0.62	0.62	0.61
$\text{corr}\left(\frac{ib_t}{y_t}, \frac{ib_{t-1}}{y_{t-1}}\right)$	0.66	0.33	0.32	0.51	0.5	0.39
$\text{corr}\left(\frac{ca_t}{y_t}, \frac{ca_{t-1}}{y_{t-1}}\right)$		0.3	0.3	0.32	0.32	
<i>Correlations with output:</i>						
$\text{corr}(c_t, y_t)$	0.59	0.94	0.94	0.84	0.85	1
$\text{corr}(i_t, y_t)$	0.64	0.66	0.66	0.67	0.67	0.66
$\text{corr}(h_t, y_t)$	0.8	1	1	1	1	1
$\text{corr}\left(\frac{ib_t}{y_t}, y_t\right)$	-0.13	-0.012	-0.013	-0.044	-0.043	0.13
$\text{corr}\left(\frac{ca_t}{y_t}, y_t\right)$		0.026	0.025	0.05	0.051	

Note: The first column was taken from Mendoza (1991). Standard deviations are measured in percent per year.

Figure 8: Moments (from SGU)

Conclusions

- ▶ The model matches unconditional correlations fairly well
 - ▶ The stationarization device has little effect on the moments
- ▶ Unconditional correlations are not that great
 - ▶ a limitation of the moment matching method?
- ▶ Correlation of consumption with output is too high
 - ▶ and probably cross-correlation of consumption too low
 - ▶ still the Backus-Kehoe-Kydland puzzle...