Small Open Economy Extension (IRBC) Macro II - Fluctuations - ENSAE, 2024-2025

Pablo Winant

2025-03-19

Introduction and Basic Facts

What are the classical reasons to open economy to trade?

trade integration

What are the classical reasons to open economy to trade?

trade integration

taste for variety

What are the classical reasons to open economy to trade?

trade integration

- taste for variety
- comparative advantage

What are the classical reasons to open economy to trade?

trade integration
 taste for variety
 comparative advantage
 financial integration

What are the classical reasons to open economy to trade?

trade integration

 taste for variety
 comparative advantage

 financial integration

 smooth shock / insurance

From RBC to IRBC

RBC models have been very successful at matching Business Cycles

- (temporary) victory against keynesian view that short term fluctuations result from demand shocks
- so successful that facts at odd with theoretical predictions have been called "puzzles"

From RBC to IRBC

RBC models have been very successful at matching Business Cycles

- (temporary) victory against keynesian view that short term fluctuations result from demand shocks
- so successful that facts at odd with theoretical predictions have been called "puzzles"

It didn't take long before the same methodology was applied to International Business Cycles

From RBC to IRBC

RBC models have been very successful at matching Business Cycles

- (temporary) victory against keynesian view that short term fluctuations result from demand shocks
- so successful that facts at odd with theoretical predictions have been called "puzzles"

It didn't take long before the same methodology was applied to International Business Cycles

Seminal Paper:

- International Real Business Cycles, Backus, Kehoe, Kydland (1992) (freshwater economists)

Very successful methodology:

facts at odd with theoretical predictions have been called "puzzles"

IRBC Facts

	Std.	Dev. %)	Rati	o of Sta to	andard that of	Devia y	ion	Autocorr. y		Corr	elation	with C	Output	
Country	у	nx	c	x	g	n	z	У	с	x	g	nx	n	z
Australia	1.45	1.23	.66	2.78	1.28	.34	1.00	.60	.46	.68	.15	01	.12	.98
Austria	1.28	1.15	1.14	2.92	.36	1.23	.84	.57	.65	.75	24	46	.58	.65
Canada	1.50	.78	.85	2.80	.77	.86	.74	.79	.83	.52	23	26	.69	.84
France	.90	.82	.99	2.96	.71	.55	.76	.78	.61	.79	.25	30	.77	.96
Germany	1.51	.79	.90	2.93	.81	.61	.83	.65	.66	.84	.26	11	.59	.93
Italy	1.69	1.33	.78	1.95	.42	.44	.92	.85	.82	.86	.01	68	.42	.96
Japan	1.35	.93	1.09	2.41	.79	.36	.88	.80	.80	.90	02	22	.60	.98
Switzerland	1.92	1.32	.74	2.30	.53	.71	.67	.90	.81	.82	.27	68	.84	.93
UK	1.61	1.19	1.15	2.29	.69	.68	.88	.63	.74	.59	.05	19	.47	.90
115	1.92	52	.75	3.27	.75	.61	.68	.86	.82	.94	.12	37	.88	.96
Europe	1.01	.52	.83	2.09	.47	.85	.98	.75	.81	.89	.10	25	.32	.85

Properties of Business Cycles in OECD Economies

Notes: Statistics are based on Hodrick-Prescott filtered data. Variables are: y, real output; c, real consumption; x, real fixed investment; g, real government purchases; nx, ratio of net exports to output, both at current prices; n, civilian employment; z, Solow residual, defined in text. Except for the ratio of net exports to output, statistics refer to logarithms of variables. Data are quarterly from the OECD's *Quarterly National Accounts*, except employment, which is from the OECD's *Main Economic Indicators*. The sample period is 1970:1 to 1990:2.

Figure 1: Moments

From Kehoe, Kydland (1995)

IRBC Facts

	1	<u>e.</u>				-		Annual I		-	-		-	
2440											- 1	10		
Sample .	1.4	1.28		2.8	1.2	12	1.96			-18	- 14	- 34	æ	
	1.0	118	1.16	1.0							- 18	1.46		
Conta -	1.00			1.0			.24		- 20	31	-3	-34		
				2.84		100						36	22	
terms.	1.4			1.0		- 44			- 44					
-	3.0	1.28		1.05			-14		- 81			- 18		
in a	1.8		1.00	3.43	- 22	- 26			- 44	-14	-4	-31		
internal.	1.67	1.4	- 24	1.8	- 22							1.48	- 84	
	1.00		1.6	1.0				.0	- 24			- 2	1.00	
1.6	1.0			10					- 40		Ð	-3	л	
	1.14		- 10	1.0	1.00							-36	-34	

Figure 2: Moments

		Correlati	on with S	ame U.S.	Variable	
Country	у	c	x	g	n	z
Australia	.51	19	.16	.23	18	.52
Austria	.38	.23	.46	.29	.47	.17
Canada	.76	.49	01	01	.53	.75
France	.41	.39	.22	20	.26	.39
Germany	.69	.49	.55	.28	.52	.65
Italy	.41	.02	.31	.09	01	.35
Japan	.60	.44	.56	.11	.32	.58
Switzerland	.42	.40	.38	.01	.36	.43
United Kingdom	.55	.42	.40	04	.69	.35
Europe	.66	.51	.53	.18	.33	.56

International Comovements in OECD Economies

Notes: See Table 1.

Figure 3: Comoments

	- M	27		-12				Annan.		-	-		-	
0440	-						1					-		
Namela	1.4	1.28		2.8	1.2	12	1.96			-14	- 14	- 34	æ	
ALC: N	1.0	118	1.10	1.0		1.28					- 18	1.46		
-	1.00			1.00			24		- 20	10	- 25	-36		
have		×.		2.00							з	-36	\mathcal{D}	
Acres 1	1.4			1.0					- 44			1.84		
-	1.0	1.10		1.00	- 44				- 81			- 18	1.00	
-	1.0		1.0	14					- 44	.4	-4	-31		1
Name of Concerns, or other	1.67	1.4	- 24	1.8	- 22							1.48	- 84	
	1.00		1.6	1.0				.0	- 24			- 2	1.00	
5.5	1.0			10					- 40		Ð	-3	л	
Test in	1.01		- 10	1.0	1.00	- 22			- 14			-36	-34	

Domestically:

output more variable than consumption

Figure 4: Moments

		Carelai	in which 2	ans U.L.	Tables.	
Crumity	2					
Antralia	.51	- 28	- 26	.25	- 18	.1
Auth		- 39	46		æ	1.8
Canada	.76		- 8	- 01	.30	.1
Transit		- 39	- 32	- 29	- 26	
Germen .			- 20	28	.10	- 4
laty	.41		31		- 81	. 3
hper	- 40		- 34		.30	- 4
Internal	.42	- 14	.38	.81	36	- 24
United Kingdom		,e	- 44	- 34	- 60	- 3
Isran		31	.33	10	39	1

Internationally:

Figure 5: Comoments

	1	<u>e.</u>						Annual I		-	-		-	
0440	-						1	-				-		
Sample .	1.4	1.28		2.8	1.2	12	1.96			-18	- 4	- 34	æ	
ALC: N	1.0	118	1.10	1.0							- 14	1.46		
Center III	1.00			1.0			.24		- 20	31	- 21	-34		
base .				2.84		100						36	22	
Acres 1	1.4			1.0		- 44			- 44			1.84		
inter la	3.0	1.28		1.01			-14		- 81			- 18		
-	1.0		1.0	14					- 44	.46	- 4	-31		
Name of Concerns, or other	1.67	1.4	- 24	1.8	- 22							1.48	- 84	
	1.00		1.6	1.0				.0	- 24			- 2	1.00	
5.5	1.0			10					- 40		Ð	-3	л	
Test in	1.01		- 10	1.0	1.00	- 22			- 14			-36	-34	

Figure 4: Moments Domestically:

- output more variable than consumption
- output autocorrelated

		Carelai	in which 2	ans U.L.	Twittle.	
Crumity	2					
Antralia	.51	- 28	- 26	.25	- 18	.1
Auth		- 39	46		æ	1.8
Canada	.76		- 8	- 01	.30	.1
Trans	.41	- 39	.32	- 20	- 26	.2
Germany .		- 48		26	.30	- 4
laty	.41		31		- 81	. 3
hper	- 40		- 34		.30	- 4
Internal	.42	- 14	.38	.81	36	- 24
United Kingdom		,e	- 44	- 34	- 60	- 3
Isran		31	.33	10	39	- 5

Internationally:

Figure 5: Comoments

	N,	22				-		Annan.		-			-	
2440	-						1	-				-		
Sample .	1.4	1.28		2.8	1.2	12	1.96			-18	- 4	- 34	æ	
	1.0	118	1.16	1.0							- 14	1.46		
Conta -	1.00			1.0			.24		- 20	31	- 21	-34		1
				2.84		100						36	22	
terms.	1.4			1.0					- 44			1.84		
-	3.0	1.28		1.05			-14		- 81			- 18		
-	1.0		1.0	14					- 44	.46	- 4	-31		
internet.	1.67	1.4	- 24	1.8	- 22							1.48	- 84	
	1.00		1.6	1.0				.0	- 24			- 2	1.00	
1.6	1.0			10					- 40		Ð	-3	л	
	1.01		- 10	1.0	1.00	- 22						-36	-34	

Figure 4: Moments

		Carelai	in which 2	ans U.L.	Twittle.	
Crumity	2					
Antralia	.51	- 28	- 26	.25	- 18	.1
Auth		- 39	46		æ	1.8
Canada	.76		- 8	- 01	.30	.1
Transit		- 39	- 32	- 29	- 26	
Germany .		- 48		26	.30	- 4
laty	.41		31		- 81	.1
hper	- 40		- 34		.30	- 4
Internal	.42	- 14	.38	.81	36	- 4
United Kingdom		,e	- 44	- 34	- 60	- 3
Internet		31	.33	10	39	- 5

Domestically:

- output more variable than consumption
- output autocorrelated
- productivity strongly procyclical

Internationally:

Figure 5: Comoments

	1	<u>e.</u>		-12		-		Annual I		-	-		-	
2440											- 64	10		
Sample .	1.4	1.28		2.8	1.2	12	1.96			-18	- 4	- 34	æ	
	1.0	118	1.16	1.0							- 14	1.46		
Conta -	1.00			1.0			.24		- 20	31	- 21	-34		
				2.84		100						36	22	
terms.	1.4			1.0		- 44			- 44			1.84		
-	3.0	1.28		1.01			-14		- 81			- 18		
in a	1.8		1.00	3.43	- 22	- 26			- 44	-14	- 4	-31		
internet.	1.67	1.4	- 24	1.8	- 22							1.48	- 84	
	1.00		1.6	1.0				.0	- 24			- 2	1.00	
1.6	1.0		- 22	1.00					- 40		Ð	-31		
	1.01		- 10	1.0	1.00	- 22			- 14			-36	-34	

Figure 4: Moments

		Carelai	in which 2	ans U.L.	Tables.	
Crumity	2					
Antralia	.51	- 28	- 26	.25	- 18	.1
Auth		- 39	46		æ	1.8
Canada	.76		- 8	- 01	.30	.1
Trans	.41	- 39	.32	- 20	- 26	.2
Germany .		- 48		26	.30	- 4
laiy	.41		31		- 81	
hper	- 40		- 34		.30	- 4
Internal	.42	- 14	.38	.81	36	- 24
United Kingdom		,e	- 44	- 34	- 60	- 3
Internet		31	.33	10	39	- 5

Figure 5: Comoments Domestically:

- output more variable than consumption
- output autocorrelated
- productivity strongly procyclical
- trade balance strongly countercyclcal

Internationally:

	м,	22				-		Annan.		-			-	
2440	-						1	-				-		
Sample .	1.4	1.28		2.8	1.2	12	1.96			-18	- 4	- 34	æ	
	1.0	118	1.16	1.0							- 14	1.46		
Conta -	1.00			1.0			.24		- 20	31	- 21	-34		1
				2.84		100						36	22	
terms.	1.4			1.0		- 44			- 44			1.84		
-	3.0	1.28		1.01			-14		- 81			- 18		
-	1.0		1.0	14					- 44	.46	- 4	-31		
internet.	1.67	1.4	- 24	1.8	- 22							1.48	- 84	
	1.00		1.6	1.0				.0	- 24			- 2	1.00	
1.6	1.0			10					- 40		Ð	-3	л	
	1.01		- 10	1.0	1.00	- 22						-36	-34	

Figure 4: Moments

		Carelai	in which 2	ans U.L.	Twittle.	
Crumity	2					
Antralia	.51	- 28	- 26	.25	- 18	.1
Auth		- 39	46		æ	1.8
Canada	.76		- 8	- 01	.30	.1
Trans	.41	- 39	.32	- 20	- 26	.2
Germany .		- 48		26	.30	- 4
laiy	.41		31		- 81	.1
hper	- 40		- 34		.30	- 4
Internal	.42	- 14	.38	.81	36	- 4
United Kingdom		,e	- 44	- 34	- 60	- 3
Internet		31	.33	10	39	- 5

Figure 5: Comoments Domestically:

- output more variable than consumption
- output autocorrelated
- productivity strongly procyclical
- trade balance strongly countercyclcal
- positive comovements in output

Internationally:

	N,	22		-12		-		Annan.		-	-		-	
2440	-						1	-				-		
Sample .	1.4	1.28		2.8	1.2	12	1.96			-18	-16	- 34	æ	
	1.0	118	1.10	1.0							- 18	- 44		
Conta -	1.00			1.0			.24		- 20	31	-3	-34		1
				2.84		100						- 34	22	
terms.	1.4			1.0		- 44			- 44		*			
-	3.0	1.28		1.01			-14		- 81			- 18		
-	1.0		1.0	14					- 44	.46	- 4	-31		
internal.	1.67	1.4	- 24	1.8	- 22							- 44	- 84	
	1.00		1.6	1.0				.0	- 24			- 21	1.00	
1.6	1.0			10					- 40		Ð	-31	л	
	1.01			1.0								- 38	-34	

Figure 4: Moments

		Carelai	in which 2	ans U.L.	Tables.	
Crumity	2					
Antralia	.51	- 28	- 26	.25	- 18	.1
Auth		- 39	46		æ	1.8
Canada	.76		- 8	- 01	.30	.1
Transit		- 39	- 32	- 29	- 26	
Germany .				28	.10	- 4
laiy	.41		31		- 81	.1
hper	- 40		- 34		.30	- 4
Internal	.42	- 14	.38	.81	36	- 4
United Kingdom		,e	- 44	- 34	- 60	- 3
Internet		31	.33	10	39	- 5

Figure 5: Comoments

Domestically:

- output more variable than consumption
- output autocorrelated
- productivity strongly procyclical
- trade balance strongly countercyclcal
- positive comovements in output
- Internationally:
 - smaller comovements in consumption

	N,	22		-12		-		Annan.		-	-		-	
2440	-						1	-				-		
Sample .	1.4	1.28		2.8	1.2	12	1.96			-18	-16	- 34	æ	
	1.0	118	1.10	1.0							- 18	- 44		
Conta -	1.00			1.0			.24		- 20	31	-3	-34		1
				2.84		100						- 34	22	
terms.	1.4			1.0					- 44		*			
-	3.0	1.28		1.05			-14		- 81			- 18		
-	1.0		1.0	14					- 44	.46	- 4	-31		
internet.	1.67	1.4	- 24	1.8	- 22							- 44	- 84	
	1.00		1.6	1.0				.0	- 24			- 21	1.00	
1.6	1.0			10					- 40		Ð	-31	л	
	1.01			1.0								- 38	-34	

Figure 4: Moments

		Carelai	in which 2	ans U.L.	Twittle.	
Crumity	2					
Antralia	.51	- 28	- 26	.25	- 18	.1
Auth		- 39	46		æ	1.8
Canada	.76		- 8	- 01	.30	.1
Trans	.41	- 39	.32	- 20	- 26	.2
Germany .				28	.10	- 4
laiy	.41		31		- 81	.1
hper	- 40		- 34		.30	- 4
Internal	.42	- 14	.38	.81	36	- 4
United Kingdom		,e	- 44	- 34	- 60	- 3
Internet		31	.33	10	39	- 5

Figure 5: Comoments Domestically:

- output more variable than consumption
- output autocorrelated
- productivity strongly procyclical
- trade balance strongly countercyclcal
- positive comovements in output
- Internationally:
 - smaller comovements in consumption
 - Backus-Kehoe-Kydland puzzle

	м,	22		-12		-		Annan.		-	-		-	
2440	-						1	-				-		
Sample .	1.4	1.28		2.8	1.2	12	1.96			-18	-16	- 34	æ	
	1.0	118	1.16	1.0							- 18	- 44		
Conta -	1.00			1.0			.24		- 20	31	-3	-34		1
				2.84		100						- 34	22	
terms.	1.4			1.0					- 44		*			
-	3.0	1.28		1.05			-14		- 81			- 18		
-	1.0		1.0	14					- 44	.46	- 4	-31		
internal.	1.67	1.4	- 24	1.8	- 22							- 44	- 84	
	1.00		1.6	1.0				.0	- 24			- 21	1.00	
1.6	1.0			10					- 40		Ð	-31	л	
	1.01			1.0								- 38	-34	

Figure 4: Moments

		Carelai	in which 2	ans U.L.	Tables.	
Crumity	2					
America	.51	- 28	- 26	.25	- 18	.1
Auth		- 39	46		æ	1.8
Canada	.76		- 8	- 01	.30	.1
Trans	.41	- 39	.32	- 20	- 26	.2
Germen .			- 20	28	.10	- 4
laiy	.41		31		- 81	.1
hper	- 40		- 34		.30	- 4
Internal	.42	- 14	.38	.81	36	- 4
United Kingdom		,e	- 44	- 34	- 60	- 3
Isran		31	.33	10	39	- 5

Figure 5: Comoments Domestically:

- output more variable than consumption
- output autocorrelated
- productivity strongly procyclical
- trade balance strongly countercyclcal
- positive comovements in output
- Internationally:
 - smaller comovements in consumption
 - Backus-Kehoe-Kydland puzzle

	N,	22		-12		-		Annan.		-	-		-	
2440	-						1	-				-		
Sample .	1.4	1.28		2.8	1.2	12	1.96			-18	-16	- 34	æ	
	1.0	118	1.16	1.0							- 18	- 44		
Conta -	1.00			1.0			.24		- 20	31	-3	-34		1
				2.84		100						- 34	22	
terms.	1.4			1.0		- 44			- 44		-			
-	3.0	1.28		1.01			-14		- 81			- 18		
-	1.0		1.0	14					- 44	.46	- 4	-31		
internet.	1.67	1.4	- 24	1.8	- 22							- 44	- 84	
	1.00		1.6	1.0				.0	- 24			- 21	1.00	
1.6	1.0			10					- 40		Ð	-31	л	
	1.01			1.0								- 38	-34	

Figure 4: Moments

		Carelai	in which 2	ans U.L.	Twittle.	
Crumity	2					
Antralia	.51	- 28	- 26	.25	- 18	.1
Auth		- 39	46		æ	1.8
Canada	.76		- 8	- 01	.30	.1
Transit		- 39	- 32	- 29	- 26	
Gener				28	.10	- 4
Jacy	.41		31		- 81	.1
hper	- 40		- 34		.30	- 4
Internal	.42	- 14	.38	.81	36	- 4
United Kingdom		,e	- 44	- 34	- 60	- 3
Incom		31	.33	10	39	- 5

Domestically:

- output more variable than consumption
- output autocorrelated
- productivity strongly procyclical
- trade balance strongly countercyclcal
- positive comovements in output
- Internationally:
 - smaller comovements in consumption
 - Backus-Kehoe-Kydland puzzle

Figure 5: Comoments

Can we replicate these moments with a BC model?

Modeling a Small Open Economy

Endowment model

Representative agents maximizes:

$$\label{eq:ct} \begin{split} \max_{c_t} \sum_{t=0}^\infty \beta^t u(c_t) \\ c_t + a_t \leq y_t + (1+r) a_{t-1} \end{split}$$

Endowment economy:

income (y_t)_t is exogenously given
 for simplicity we assume it is deterministic

Small open economy:

open: can save a_t which yields a_{t+1}(1+r) in the next period
 small: country takes world interest rate r as given (no effect on world prices)

We solve this problem with the terminal conditions:

•
$$a_{-1}$$
 given¹

We get the lagrangian:

$$\mathcal{L} = \sum_{t=0}^\infty \beta^t u(c_t) + \sum_{t=0}^\infty \beta^t \lambda_t \left(y_t + (1+r)a_{t-1} - c_t - a_t\right)$$

First order conditions:

$$c_0 = \frac{r}{1+r} \left\{ (1+r)a_{-1} + \sum_{t=0}^\infty \frac{y_t}{(1+r)^t} \right\}$$

We get the lagrangian:

$$\mathcal{L} = \sum_{t=0}^\infty \beta^t u(c_t) + \sum_{t=0}^\infty \beta^t \lambda_t \left(y_t + (1+r)a_{t-1} - c_t - a_t\right)$$

First order conditions:

$$c_0 = \frac{r}{1+r} \left\{ (1+r)a_{-1} + \sum_{t=0}^\infty \frac{y_t}{(1+r)^t} \right\}$$

We get the lagrangian:

$$\mathcal{L} = \sum_{t=0}^\infty \beta^t u(c_t) + \sum_{t=0}^\infty \beta^t \lambda_t \left(y_t + (1+r)a_{t-1} - c_t - a_t\right)$$

First order conditions:

$$c_0 = \frac{r}{1+r} \left\{ (1+r)a_{-1} + \sum_{t=0}^\infty \frac{y_t}{(1+r)^t} \right\}$$

We get the lagrangian:

$$\mathcal{L} = \sum_{t=0}^\infty \beta^t u(c_t) + \sum_{t=0}^\infty \beta^t \lambda_t \left(y_t + (1+r)a_{t-1} - c_t - a_t\right)$$

First order conditions:

$$c_0 = \frac{r}{1+r} \left\{ (1+r)a_{-1} + \sum_{t=0}^\infty \frac{y_t}{(1+r)^t} \right\}$$

Current Account



The **trade balance** is exports-imports (here $y_t - c_t$) The **current account** is trade balance + net factor payments (here $y_t - c_t + ra_{t-1}$) Positive **current account**: additional lending to the rest of the world.

Current Account

Reminders on Current Account

The **trade balance** is exports-imports (here $y_t - c_t$) The **current account** is trade balance + net factor payments (here $y_t - c_t + ra_{t-1}$) Positive **current account**: additional lending to the rest of the world.

Using the formula from before

$$CA_0 = a_{-1}r + (1 - \frac{r}{1+r})y_0 - \frac{r}{1+r} \left\{ \sum_{t \ge 1}^\infty \frac{y_t}{(1+r)^t} \right\}$$

How does the current account reacts to income shocks?



> and to news about future income shocks:

Unit root

Still with the same formula:

$$c_0 = \frac{r}{1+r} \left\{ (1+r)a_{-1} + \sum_{t=0}^\infty \frac{y_t}{(1+r)^t} \right\}$$

What is the effect of an increase in a_{-1} ?

- consumption rises permanently
- \$a_t\$ is constant, equal to \$a_{-1}\$
- agent consumes small amount \$r\$ corresponding to interest
 - this will correspond to a unit root in the solution

Adding capital

We add capital and production to our endowment economy:

$$y_t = z_t k_{t-1}^\alpha$$

$$k_t = (1-\delta)k_{t-1} + i_t$$

The aggregate resource constraint becomes:

$$a_t + c_t + i_t = (1+r)a_{t-1} + y_t$$

Now maximize $\sum_t \beta^t U(c_t)$

Adding capital

We add capital and production to our endowment economy:

$$y_t = z_t k_{t-1}^\alpha$$

$$k_t = (1-\delta)k_{t-1} + i_t$$

The aggregate resource constraint becomes:

$$a_t+c_t+i_t=(1+r)a_{t-1}+y_t$$

Now maximize $\sum_t \beta^t U(c_t)$

We get first order conditions

$$\begin{split} \lambda_t &= \beta \lambda_{t+1} (1+r) \\ \lambda_t &= \beta \lambda_{t+1} \left[(1-\delta) + z_{t+1} f'(k_t) \right] \end{split}$$

where λ_t is lagrange multiplier associated to budget constraint.

Adding capital: optimality conditions

Since $\lambda_t > 0$ (constraint is always binding), we get:

$$(1-\delta) + z_{t+1}f'(k_t) = 1 + r$$

$$k_t = \left(\frac{r+\delta}{\alpha z_{t+1}}\right)^{\frac{1}{\alpha-1}}$$

and investment

$$i_t = \left(\frac{r+\delta}{\alpha z_{t+1}}\right)^{\frac{1}{\alpha-1}} - (1-\delta) \left(\frac{r+\delta}{\alpha z_t}\right)^{\frac{1}{\alpha-1}}$$

Adding capital: optimality conditions

Since $\lambda_t > 0$ (constraint is always binding), we get:

$$(1-\delta) + z_{t+1}f'(k_t) = 1 + r$$

$$k_t = \left(\frac{r+\delta}{\alpha z_{t+1}}\right)^{\frac{1}{\alpha-1}}$$

and investment

$$i_t = \left(\frac{r+\delta}{\alpha z_{t+1}}\right)^{\frac{1}{\alpha-1}} - (1-\delta) \left(\frac{r+\delta}{\alpha z_t}\right)^{\frac{1}{\alpha-1}}$$

Here investment is fully determined by productivity shocks

too simple: no international dependence

Add friction to the investment

A possible solution: change the resource constraint such that adjusting capital is costly

For instance:

$$\begin{split} a_t + c_t + i_t + \frac{\omega}{2} \frac{(k_t - k_{t-1})^2}{k_t} &= (1+r) a_{t-1} + z f(k_{t-1}) \\ k_t &= (1-\delta) k_{t-1} + i_t \end{split}$$

where ω is an adjustment friction.

Add friction to the investment

A possible solution: change the resource constraint such that adjusting capital is costly

For instance:

$$a_t + c_t + i_t + \frac{\omega}{2} \frac{(k_t - k_{t-1})^2}{k_t} = (1+r)a_{t-1} + zf(k_{t-1})$$

$$k_t = (1-\delta)k_{t-1} + i_t$$

where ω is an adjustment friction.

Typically, ω is chosen so that the model replicates $\frac{Var(i_t)}{Var(y_t)}$ from the data.

Add friction to the investment

A possible solution: change the resource constraint such that adjusting capital is costly

For instance:

$$a_t + c_t + i_t + \frac{\omega}{2} \frac{(k_t - k_{t-1})^2}{k_t} = (1+r)a_{t-1} + zf(k_{t-1})$$

$$k_t = (1-\delta)k_{t-1} + i_t$$

where ω is an adjustment friction.

Typically, ω is chosen so that the model replicates $\frac{Var(i_t)}{Var(y_t)}$ from the data.

Cf tutorial.

A benchmark Small Open Economy Model

A benchmark Small Open Economy Model



Stephanie Schmitt-Grohe and Martin Uribe.

Figure 6: Stephanie Schmitt Grohe and Martin Uribe

Closing Small Economy Models, Schmitt Grohe and Uribe (2003), JIE

- small open economy model with production, consumption-leisure tradeoff and capital adjustment costs
 = RBC+open+adj costs
- perform some moments matching
- compare different ways of stationarizing

The model

$$\max_{c_t,n_t}\sum_{t=0}^\infty \beta^t u(c_t,n_t)$$

$$\begin{split} c_t + k_t + a_t &= y_t + g_t - \frac{\omega}{2} (k_t - k_{t-1})^2 + (1 - \delta) k_{t-1} + (1 + r^\star + \pi(a_{t-1})) a_{t-1} \\ y_t &= f(k_{t-1}, n_t, z_t) \end{split}$$

$$z_{t+1} = \rho z_t + \epsilon_{t+1}$$

 and

$$u(c,n) = \frac{1}{1-\sigma} \left(c^{\psi} (1-n)^{1-\psi}) \right)^{1-\sigma}$$

The model

$$\max_{c_t,n_t} \sum_{t=0}^\infty \beta^t u(c_t,n_t)$$

$$\begin{split} c_t + k_t + a_t &= y_t + g_t - \frac{\omega}{2} (k_t - k_{t-1})^2 + (1 - \delta) k_{t-1} + (1 + r^\star + \pi(a_{t-1})) a_{t-1} \\ & y_t = f(k_{t-1}, n_t, z_t) \end{split}$$

$$z_{t+1} = \rho z_t + \epsilon_{t+1}$$

and

$$u(c,n) = \frac{1}{1-\sigma} \left(c^{\psi} (1-n)^{1-\psi}) \right)^{1-\sigma}$$

The term π is there to make the model stationary.

How to make the distribution stationary?

The solution of the model exhibits a unit root:

 $a_t = a_{t-1} + \dots$ other variables in t-1 + shocks in t

How to make the distribution stationary?

The solution of the model exhibits a unit root:

 $a_t = a_{t-1} + \ldots {\rm other}$ variables in t-1 + shocks in t

Problem:

- there isn't a unique deterministic steady-state
- the ergodic distribution of the model variables is not defined

This raises practical issues (notably for estimation) for the *linear* model.

no unconditional moments

How to get rid of the unit root?

General idea:

introduce a force that pulls the level of foreign assets towards equilibrium

Schmitt Grohe and Uribe (2003) consider many options:

debt-elastic interest rate:

$$1+r=1+r^\star+\pi(a_d)$$

• with $\pi(0) = 0$ and $\pi'(0) > 0$ • π can be understood as a risk premium on rising debt

endogenous time-discount (aka Usawa preferences)

$$\beta(c_t) = (1+c_t)^{-\chi}$$

costs of adjustment for international portfolios

How to get rid of the unit root?

General idea:

introduce a force that pulls the level of foreign assets towards equilibrium

Schmitt Grohe and Uribe (2003) consider many options:

debt-elastic interest rate:

$$1+r=1+r^\star+\pi(a_d)$$

with π(0) = 0 and π'(0) > 0
 π can be understood as a risk premium on rising debt

endogenous time-discount (aka Usawa preferences)

$$\beta(c_t) = (1+c_t)^{-\chi}$$

costs of adjustment for international portfolios

SGU show that the choice of the stationarization device has little effect for the dynamics (moments) of most variables

Calibration

Parameters	Values	Parameters	Values
σ	2	$\overline{\delta}$	0.1
ψ	1.45	ρ	0.42
α	0.32	σ^2	0.0129
ω	0.028	A^{\star}	-0.7442
r	0.04	χ	0.000742

Results



Fig. 1. Impulse response to a unit technology shock in Models 1–5. Note. Solid line: Endogenous discount factor model; Squares: Endogenous discount factor model without internalization; Dashted line: Debt-elastic interest rate model; Dash-dotted line: Portfolio adjustment cost model; Dotted line: complete asset markers model; Circles: Model without stationarity inducing elements.

Figure 7: Impulse Response Function

Table 3 Observed and implied second moments

	Data	Model 1	Model 1a	Model 2	Model 3	Model 4
Volatilities:						
std(y,)	2.8	3.1	3.1	3.1	3.1	3.1
std(c,)	2.5	2.3	2.3	2.7	2.7	1.9
std(i,)	9.8	9.1	9.1	9	9	9.1
std(h _i)	2	2.1	2.1	2.1	2.1	2.1
$\operatorname{std}\left(\frac{tb_i}{y_i}\right)$	1.9	1.5	1.5	1.8	1.8	1.6
$\operatorname{std}\left(\frac{ca_i}{y_i}\right)$		1.5	1.5	1.5	1.5	
Serial correlations:						
$corr(y_i, y_{i-1})$	0.61	0.61	0.61	0.62	0.62	0.61
$corr(c_i, c_{i-1})$	0.7	0.7	0.7	0.78	0.78	0.61
$corr(i_0, i_{i-1})$	0.31	0.07	0.07	0.069	0.069	0.07
$corr(h_{e}, h_{e-1})$	0.54	0.61	0.61	0.62	0.62	0.61
$\operatorname{corr}\left(\frac{tb_{t}}{y_{t}}, \frac{tb_{t-1}}{y_{t-1}}\right)$	0.66	0.33	0.32	0.51	0.5	0.39
$\operatorname{corr}\left(\frac{ca_{t}}{y_{t}}, \frac{ca_{t-1}}{y_{t-1}}\right)$		0.3	0.3	0.32	0.32	
Correlations with ou	utput:					
$corr(c_i, y_i)$	0.59	0.94	0.94	0.84	0.85	1
corr(i,, y,)	0.64	0.66	0.66	0.67	0.67	0.66
$corr(h_i, y_i)$	0.8	1	1	1	1	1
$\operatorname{corr}\left(\frac{tb_i}{y_i}, y_i\right)$	-0.13	-0.012	-0.013	-0.044	-0.043	0.13
$\operatorname{corr}\left(\frac{ca_{i}}{y_{i}}, y_{i}\right)$		0.026	0.025	0.05	0.051	

Note. The first column was taken from Mendoza (1991). Standard deviations are measured in percent per year.

Figure 8: Moments (from SGU)

Conclusions

The model matches unconditional correlations fairly well
 The stationarization device has little effect on the moments
 Unconditional correlations are not that great

 a limitation of the moment matching method?

 Correlation of consumption with output is too high

 and probably cross-correlation of consumption too low
 still the Backus-Kehoe-Kydland puzzle...