A Model of External Debt and International Reserves

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Abstract

The literature has addressed the issue of choosing debt and reserves levels in the context of models of debt repudiation. The focus has thus been on the effect of the balance sheet composition immediately after a default and the subsequent closure of capital markets. In practice, for developing countries, shocks to the balance of payments are mostly exogenous. Indeed, using panel data for 138 low-income and middle-income countries over the period 1967-2012, we find that central bank reserves are most often used after terms of trade shocks. Hence, we model the optimal asset-liability decisions of a country assuming it faces risks of exogenous external shocks. External borrowing finances a scale-up in public capital. The optimal levels of external debt and of the stock of capital are functions of the productivity of capital, of the cost of financing, and of the welfare losses generated by external shocks. These losses depend on the probability and the extent of a balance of payments crisis, on risk aversion, and on short-term debt. Reserves are accumulated to reduce the welfare losses. However, because external debt is endogenous, the choice of reserves and of external debt is a joint decision. In particular, additional reserves reduce the cost of a crisis and increase the level of debt, of the capital stock, and of income.

Keywords: debt ; public investment; international reserves ; rollover risk

JEL Classification: F32; F34 ; F41

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1 Introduction

Following a decade of robust growth and debt relief initiatives, many developing countries now face significantly lower debt burdens and have envisaged large scale investment projects financed by fresh borrowing. This push can be justified by the investment needs that these countries face, but the willingness of many countries to scale up spending has brought back discussions, especially in the lenders' community, of the risks that large investment and new debt could generate if growth failed to pick up, repeating historical patterns of high debt, low growth, and vulnerability to external shocks. From the developing countries perspective, the growth prospects, low levels of debt and relative resilience to the global crisis of 2008-2009 justify the ambitious strategies, but the international community, in particular official creditors, have usually relied on conservative frameworks to assess macroeconomic sustainability. For instance, the Debt Sustainability Analysis of the IMF and the World Bank, which is used, most importantly, to assess the viability of fiscal policy, estimates the risk of debt distress using simple debt ratios such as debt/GDP and debt/exports. This framework has been criticized for not being built on coherent models of growth, fiscal spending, and debt risk (Wyplosz, 2007; Eaton, 2002), and the IMF has recently built fully-fledged macroeconomic models to strengthen such analyses and take into account, in particular, the growth benefits of public spending on capital goods (Buffie et al. 2012, Andrle et al. 2011).

However, these macroeconomics models are not built to discuss the asset-liability choices that are in fact inherent to different strategies of external debt and public investment. This is the gap we intend to fill in this paper. We model the optimal asset-liability management decisions of a developing country's government with investment needs and facing risks of external shocks.

We build a model of sovereign debt where external borrowing finances investment. But a country can be hit by adverse external shocks that increase gross financing needs — either because the current account deficit is widened (the most common source of shock for Low Income Countries (LICs) and Lower-Middle Income Countries, especially for those without market access) or because the financial account is affected (the largest shocks for emerging markets and LICs with market access). Investment is thus risky from a macroeconomic point of view, because it is financed by debt which is subject to roll-over risk. The model solves the risk-return tradeoff that lies behind the investment decision. In particular, we can show, in a 3-period version of the model, how debt service, risk aversion and the probability and extent of a crisis affects the minimal internal rate of return that should be accepted when choosing the level of public investment.

The model also allows us to discuss the role of international reserves. Reserves have usually been seen as buffers against external shocks, but the accumulation of reserves for mercantilist purposes (as estimated, e.g. by Durdu et al. 2009) has changed the terms of the discussion in several large emerging markets. The situation has been different for low income countries, however, since the median reserve coverage ratio in LICs is moderate (around 4.7 months of imports) and a quarter of LICs still hold reserves below three months of imports (IMF, 2011). Although traditional metrics such as reserves/months of imports, or reserves/short-term debt or monetary base, are appealing because simple ratios are easy to present and permit comparison across countries, the need to assess reserves in analytical frameworks has generated interest even in the LIC-centric literature, with recent proposals by Barnichon (2009) and Dabla-Norris et al. (2012).

For those many countries where external debt is mostly government debt, accumulating reserves is an external balance sheet operation that the general government controls entirely. For a given net International Investment Position (IIP), reserve accumulation is performed by borrowing debt in excess of what is needed to finance deficits. When borrowing is on non-concessionary terms, international reserves are costly because the rates of return on reserves are lower than the interest rate charged on (marginal) external borrowing. In standard precautionary savings models, the joint issuance of debt and the accumulation of reserves would thus seem to be a puzzle. Indeed, the models of reserve accumulation as precautionary savings in Jeanne (2007) or in Barnichon (2009), discussed in more detail below, assume that reserves are the sole financial instrument available. In these papers, reserves are accumulated because countries want to attain a positive net wealth.

In contrast, the model presented in this paper is a model of balance sheet operations. External debt and international reserves co-exist because of different risk-return characteristics. More precisely, reserves substitute for financing, which is unavailable at short horizons in crisis times. The optimal level of reserves is found to be a function of short-term debt (as in the 'Greenspan-Guidotti Rule'), of the shock to consumption, and of the interest rate differential between external debt and reserves. But because reserves help insure against negative shocks, they also reduce the risk of taking on additional external debt. We therefore show how a reserves buffer affects the optimal level of debt and public investment. The 3-period model is solved analytically, providing the main intuitions. An infinite horizon model is next calibrated to a typical developing country and solved recursively.

The paper is organized as follows. Section 2 critically surveys the literature, discussing the relevance of the existing research to the problem faced by developing countries. Section 3 presents some evidence on the frequency of reserve usage. The 3-period model is presented and solved in Section 4, whereas the infinite horizon model is discussed in Section 5. Section 6 concludes.

2 Related Literature

The literature on accumulation of central bank reserves has taken two routes. An optimal control literature, started by Frenkel and Jovanovic (1981), focused on the short-term operation of stabilizing capital flows and exchange rates and modeled the use and accumulation of central bank reserves as an inventory problem. A recent example in this literature include Bar-Ilan et al. (2007).

The macro-economic literature, which is more relevant to this paper, has recognized the role of central bank reserves in smoothing consumption when a country is hit by external shocks. The empirical literature in LICs, for instance, has shown that countries with reserves coverage higher than three months of imports were better able to support consumption (Crispolti and Tsibouris, 2012). Reserves were also found to support consumption and real-per-capita investment during the global crisis of 2008-2009 (IMF, 2012).

The macro-economic theory literature on central bank reserves initially borrowed from models of bank runs. Aizenman and Lee (2005)'s model is an application of the Diamond-Dybvig (1983) framework: a country is similar to a deposit-taking bank that can be hit by funding shocks. When a liquidity shock hits, reserves reduce the need to liquidate assets at short notice, thereby reducing output losses. Aizenman and Lee (2005) solve different specifications of the model. When reserves are costless, the optimal strategy is to take as much (net) debt as possible in order to maximize output whist borrowing enough reserves to offset any liquidity shock. More generally, the optimal holding of reserves is such that the opportunity cost of holding reserves is equal to its expected precautionary benefit (which are directly related to the liquidation costs). The model is a two-period model in which decision makers are risk neutral, and the only effect of liquid assets is to prevent liquidation and reduce output losses.

Jeanne (2007) and Jeanne and Ranciere (2011) provide models of reserves accumulation that are closer in spirit to ours. Although they do not discuss optimal liabilities, the focus is explicitly on consumption-smoothing. In Jeanne and Ranciere (2011), countries issue a long-term bond to accumulate reserves. When a sudden stop cuts off the economy from international capital market, the country is forced to repay its short-term debt, and GDP declines exogenously. To help ease the pain, countries default on their long-term debt, and reserves are used to fill the balance of payments gap. Optimal reserves formula capture the Guidotti rule: reserves should cover short-term external liabilities in addition to finance imports that ensure a stable level of consumption. However, to some extent, it is the choice of default, and not the accumulation of reserves, that helps transfer resources across states of nature — allowing the strategy to work as an optimal insurance.

Alfaro and Kanczuk (2009) and Bianchi et al (2013) present dynamic games models in the tradition of Eaton and Gersovitz (1981). Borrowing countries default on their external liabilities when the financial gains from defaulting exceed the welfare losses incurred by the closure of international markets after default. Although reserves could help finance consumption when the country is cut from capital markets, Alfaro and Kanczuk (2009) find that the optimal level reserves is 0. Indeed, having no reserves increases the cost of defaulting and therefore credibly "ties the hand" of the borrower. With a lower probability of default, interest rates on foreign loans are lowered, which is why a zero-reserve policy is optimal. This result is counter-intuitive, but Bianchi et al (2013) shows that some reserves are held in a similar model that also takes into account the maturity of debt. Overall, this literature assumes that reserves are used as financing when countries default, an assumption that is at odds with the practical policy of countries to hold reserve in order to weather exogenous shocks, not necessarily attributed to default.

The literature on Low Income Countries has been less extensive. The papers surveyed focused on sudden stops, default, or were motivated by the accumulation of reserves in large emerging narkets. An exception is Barnichon (2009), who applies a precautionary savings model to LICs facing risks of hurricanes and droughts. A weakness of the model is that is really a model of net asset position, not a model of reserves, and it suggests that LICs should be net savers. Carroll and Jeanne (2010) model is built along the same lines.

Buffie et al. (2011) have focused on the other side of the balance sheet of LICs. They develop a model-based Debt Sustainability Framework (DSF) in which debt dynamics are integrated to a consistent modern macroeconomic model. The dynamic interactions between spending, growth, and future revenues drive debt dynamics. Although the model captures several of the challenges to public investment in LICs, in particular the 'inefficiency' of public investment, the model is not set to discuss *risks* and mitigating strategies — although alternative scenarios can be constructed to assess the sensitivity of dynamics to different parameter assumptions.

3 When do central bank draw on their international reserves?

The empirical literature has shown that the buffer provided by central bank reserves is useful to smooth consumption, limit output drop, and even reduce the probability of sudden stops in emerging markets/Middle-Income Countries (MICs) as well as in LICs (Crispolti and Tsoubiris, 2012; Jeanne and Ranciere, 2011). However, less is known about the occasions in which central bankers indeed tap into reserves. This is an important question because theoretical models are designed around a specific risk scenario where reserves can be put to use. Our discussion of the theoretical literature noted that most models for emerging markets assumed that reserves are used following a default on external debt (e.g. Jeanne and Ranciere, 2011), which is why "tying one's hand" with zero reserve accumulation might actually be a positive signal to private creditors and thus be the optimal policy to achieve low interest rates (Alfaro and Kanczuk, 2009 ; Bianchi et al., 2013). Alternatively, Barnichon (2009)'s model, which focuses on exogenous shocks, was calibrated assuming reserves are used following natural disasters.

This section shows in fact that, for both LICs with no market access as well as MICs with market access, reserves are most typically used when the country is facing terms of trade shocks (which are exogenous for small countries). The use of reserves following sovereign default is in fact uncommon, and default or debt restructuring events are not statistically significant predictors of large drops in reserves. Similarly, natural disasters do not predict large usages of reserves.

3.1 Data

We collect annual data on reserves coverage (expressed in months of imports) for 138 developing countries (see Appendix 1 for a list of countries and of data sources) since 1967. Drops in reserves are defined as events when either: (i) the de-trended reserve coverage ratio is below its country-specific 20th percentile, and the annual change in reserves is below the (global) 20th percentile (which is -0.64 months of imports); [1] or (ii) reserve coverage falls, year-on-year, by more than 1.2 month of imports (which is the the global 10th percentile). Such identification of events is common when assessing large changes for stock variables (for instance Gourinchas et al. 2001, Mendoza and Terrones, 2008, use a similar strategy to identify credit booms, and Ghosh et al. (2012) use it for identifying capital flow surges). The strategy allows us to capture declines in reserves that present a large and economically

[1] The trend is computed for each country using an HP filter with smoothing parameter 6.25, as recommended by Ravn and Uhlig (2005). The 20th percentile is computed country-by-country

significant variation relative to the average variability of the time series of each country (part (i)) or large variations in an absolute sense (part (ii)).

We next identify external shocks with a similar methodology and the same percentile thresholds, using balance of payments data on (i) the terms of trade, (ii) FDI net inflows, and (iii) official flows, defined as the sum of official transfers and official financing.[2] We will check the robustness of our results to an alternative identification strategy, in which we first identify large foreign exchange losses for the sum: trade balance[3] + official flows + FDI (using a 20th percentile country-specific threshold and the 10th percentile global threshold, equal to -6.5 percent of GDP); and then use the same thresholds, in percent of GDP, for the subcomponents trade balance, official flows, and FDI.

In addition, we identify large natural disasters using the EMDAT database. The 10 percentile worst natural disasters, i.e. disasters than affected more than 7 percent of the population of a country, are defined as large natural disasters. Finally, we look at several measures of economic crisis events. Cruces and Trebesch (2013) and Das et al. (2010) collected data on sovereign debt restructuring events with both private creditors and official (Paris Club) creditors. This dataset covers a wider range of countries than the dataset of debt defaults provided by Reinhart and Rogoff (2011), and includes in particular many Low Income Countries. On the other hand, debt restructuring sometimes precede or follow default events, which is why we use both datasets to investigate whether debt distress is related to drawing down of international reserves.

3.2 Stylized facts

Table 1 summarizes the data. The last row shows the frequency with which central banks have reduced sharply their holdings of international reserves. On average, reserves are reduced for around 20 percent of country-years observations. In these occasions, 47 percent of the time, the reduction in reserves was concomitant with a balance of payments shocks — most frequently shocks to official financing and shocks to the terms of trade (for LICs). Only rarely were reserves declines associated to private debt restructuring or to external debt crises. These events are indeed much less frequent than large reductions in reserves: for instance, the dataset of Das et al. (2010) only contains private 190 debt restructurings. For LICs, debt restructuring with the Paris Club was sometimes contemporaneous to a decline in reserves (9 percent for LICs), but in half of these events the balance of payment was also hit, shedding doubts on the interpretation of this statistics.

Reserves declines were more often associated to currency crisis for Middle-Income Countries with a floating exchange rate — almost by definition. External debt crises were sometimes contemporaneous to large reserve usage in MICs, but not often in LICs. We will need an estimated model to she light on the significance of these raw numbers.

[2]For the terms of trade, a shock is defined as a period when the terms of trade (i) fall by more than 10 percent; or (ii) fall below 1.5 standard deviation and below 5 percent. For FDI, the global 20th percentile threshold is -0.75 percent of GDP and the 10th percentile is -2.12 percent of GDP. For official flows, the 20th and 10th percentiles are, respectively, -1.2 and -3.1 percent of GDP.

[3]the computation assumes constant volumes and exports and imports; thus any change in the trade balance can be attributed to terms of trade shocks

Table 1: Summary statistics: frequency of shock at times when reserves are used

	Low Income Countries			Middle Income Countries		
Freq. of shocks	All	Floating ExR	Fixed ExR	All	Floating ExR	Fixed ExR
Bal. of payt. shock	0.47	0.52	0.45	0.41	0.38	0.45
Terms of trade shock	0.28	0.34	0.29	0.20	0.25	0.25
FDI shock	0.12	0.06	0.11	0.13	0.08	0.13
Official fin. shock	0.17	0.23	0.16	0.13	0.09	0.13
Private debt restructuring	0.01	0.02	0.01	0.03	0.06	0.03
excl periods of bal. of payt. shock	0.01	0.01	0.01	0.02	0.05	0.01
Paris Club debt restructuring	0.09	0.16	0.09	0.04	0.06	0.02
excl periods of bal. of payt. shock	0.05	0.09	0.06	0.02	0.03	0.01
Currency crisis	0.05	0.14	0.01	0.16	0.42	0.00
excl periods of bal. of payt. shock	0.03	0.08	0.01	0.09	0.24	0.00
Domestic debt crisis	0.00	0.00	0.00	0.03	0.05	0.05
excl periods of bal. of payt. shock	0.00	0.00	0.00	0.02	0.02	0.02
External debt crisis	0.07	0.10	0.07	0.09	0.22	0.07
excl periods of bal. of payt. shock	0.04	0.02	0.04	0.05	0.12	0.02
Banking crisis	0.04	0.09	0.05	0.10	0.21	0.08
excl periods of bal. of payt. shock	0.03	0.06	0.03	0.05	0.12	0.04
Natural disaster	0.14	0.14	0.08	0.10	0.12	0.06
excl periods of bal. of payt. shock	0.09	0.08	0.06	0.07	0.09	0.03
Number of Obs.	2036	547	799	2581	806	819
Obs. when reserves are used	444	125	160	575	191	163
Freq. of reserve usage	0.22	0.23	0.20	0.22	0.24	0.20

3.3 Econometric estimations

We now estimate a variety of logit and probit models to shed light on the significance of the determinants behind reserves usage (see Table 2). The first column shows that for a logit model with fixed effects, terms of trade socks and shocks to official financing significantly increase the probability of drawing down reserves. Somewhat surprisingly, natural disasters do not increase the probability of drawing down reserves. Debt restructuring with private creditors is not a significant predictor of usage of reserves. Debt restructuring under the Paris Club actually reduces the probability of using reserves, certainly because this form of restructuring reduces the debt burden without affecting access to further financing.

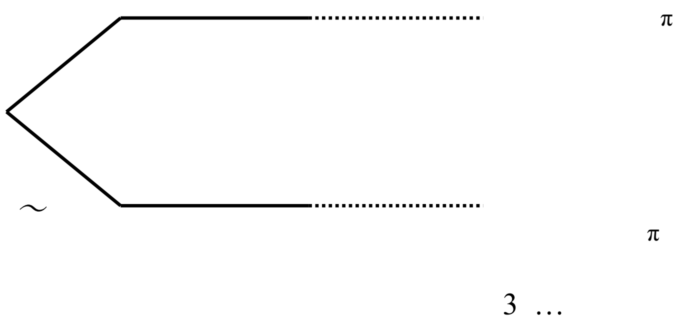
Past shocks are not related to reserve declines — presumably because central bank reserves can be used immediately, and thus reserve declines are either contemporaneous or never happen. The results are robust to different specifications (probit models, random effects). For completeness and as an exercise in robustness, two models taking into account all the crisis data from Reinhart and Rogoff (2011) are also estimated in columns 5 and 6. The main results are unchanged, although official financial shocks loose in significance; this is mostly because the dataset from Reinhart and Rogoff (2011) does not cover many LICs. The indicator for currency crisis is a strong predictors of reserve usage, but this is an endogenous variable. Debt crises and banking crises do not help predict reserve usage. Finally, Table 5 (in Appendix 1) shows that there are no significant differences between countries with a fixed or with a flexible exchange rate regime, or between LICs and MICs. The results are also mostly unchanged when using the alternative identification strategy presented above (where the thresholds for identifying shocks are kept constant across items of the balance of payments) – see Table 6 in Appendix 1.

Table 2: Logit and Probit models

Variables	(1) logit pooled	(2) probit pooled	(3) logit fixed effects (FE)	(4) logit fixed effects (FE)	(5) logit fixed effects (FE)	(6) logit fixed effects (FE)
Terms of trade shock	0.568***	0.336***	0.585***	0.628***	0.669***	0.643***
	[6.427]	[6.340]	[6.530]	[7.027]	[4.393]	[4.283]
Terms of trade shock (t-1)	0.122	0.0719	0.150			
	[1.297]	[1.303]	[1.573]			
FDI shock	-0.127	-0.0743	-0.117	-0.0676	-0.0379	-0.102
	[-1.190]	[-1.209]	[-1.062]	[-0.617]	[-0.183]	[-0.506]
FDI shock (t-1)	-0.187*	-0.108*	-0.173			
	[-1.715]	[-1.730]	[-1.556]			
Official fin. shock	0.141	0.0818	0.176*	0.212**	0.0923	0.0986
	[1.371]	[1.363]	[1.663]	[2.017]	[0.433]	[0.472]
Official fin. shock (t-1)	-0.00798	-0.00843	0.0278			
	[-0.0764]	[-0.139]	[0.256]			
Natural disaster	-0.105	-0.0608	-0.110	-0.0935	-0.411*	-0.364*
	[-0.939]	[-0.939]	[-0.886]	[-0.761]	[-1.827]	[-1.696]
Natural disaster (t-1)	-0.00445	-0.00499	-0.00951			
	[-0.0402]	[-0.0773]	[-0.0780]			
Debt restruct. (with priv. creditors)	-0.156	-0.0882	-0.199	-0.182	0.0323	-0.000227
	[-0.668]	[-0.666]	[-0.842]	[-0.778]	[0.0990]	[-0.000717]
Debt restruct. (with priv. creditors) (t-1)	-0.0320	-0.0220	-0.0751			
	[-0.145]	[-0.173]	[-0.331]			
Debt restruct. (with Paris Club)	-0.366**	-0.205**	-0.342**	-0.320**	-0.390	-0.280
	[-2.457]	[-2.446]	[-2.237]	[-2.111]	[-1.466]	[-1.094]
Debt restruct. (with Paris Club) (t-1)	-0.0757	-0.0442	-0.0639			
	[-0.549]	[-0.553]	[-0.441]			
Currency crisis					0.727***	
					[4.574]	
Domestic debt crisis					0.127	
					[0.335]	
External debt crisis					-0.0912	
					[-0.463]	
Banking crisis					0.207	
					[1.239]	
Currency crisis (t-1)						-0.268
						[-1.577]
Domestic debt crisis (t-1)						0.414
						[1.101]
External debt crisis (t-1)						0.116
						[0.631]
Banking crisis (t-1)						-0.159
						[-0.901]
Constant	-1.279***	-0.779***				
	[-22.62]	[-23.87]				
Observations	4,479	4,479	4,476	4,613	1,604	1,604
Number of countries			137	137	44	44

Robust z-statistics in brackets
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Figure 1: Time line



4 Three-period model

We now focus on the theoretical determinants of external debt and international reserves. The model is built along a single agent's consumption problem. The agent is a benevolent general government, which is the main external borrower for the countries we seek to model. The budget constraints can be thought of either those of the government or those of the country with appropriate simplifications (in particular, if the budget constraints represent the country's ones, the main assumptions would be that the private sector cannot borrow externally, that private capital adjusts without friction to public capital, and that there is only one sector and one type of household).

The general government is a composite agent that borrows externally, invests in capital goods, and accumulate reserves. The central bank, which accumulates reserves, is a separate administration and in some countries coordination between central bank and government may be weak. However, for many LICs, discussions on the macroeconomic strategy are comprehensive and decisions on financing, reserves accumulation and fiscal policy are centralized (for instance under an IMF/World Bank program).

4.1 Model

The timeline of the model is presented in Figure 1. In period 0, the agent decides how much to borrow (b), how much to invest in the capital stock (K) and how much to accumulate in reserves R. Uncertainty is then resolved, with two possible states of nature. In the *unconstrained* state of nature U (upper branch in Figure 1), output is high, there are no external shocks, and external financing is available at an interest rate r. In the *constrainted* state of nature C, which occurs with probability π, the country suffers from a 'sudden stop' in period 1: (i) output falls exogenously; (ii) a fraction m of external debt is due; (iii) and an additional foreign exchange shortfall of $\epsilon < 0$ is suffered by the country. In addition, the country is liquidity constrained, i.e. it cannot borrow. In period 2, output recovers and access to external financing is restored (periods 2 to ∞ are all identical). The maximization problem can therefore be written as follows

$$\max_{b,K,(c_i^C,c_i^U,d_i^U)_{i\geq 1},(d_i^C)_{i\geq 2}} (1-\pi)\left[u(c_1^U)+\beta(c_2^U)+\sum_{i\geq 3}\beta^{i-1}u(c_i^U)\right]+\pi\left[u(c_1^C)+\beta u(c_2^C)+\sum_{i\geq 3}\beta^i u(C_i^C)\right]$$

subject to (the lagrangian multipliers are shown in brackets for their respective budget constraints):

$$c_1^U = f_1^U(K)+d_1^U-(m+r)(b+R)+(1+r^*)R \qquad (\varphi_1) \qquad (1)$$
$$c_2^U = f_2^U(K)+d_2^U-(1+r)d_1^U-(1-m)(1+r)(b+R) \qquad (\varphi_2) \qquad (2)$$
$$c_i^U = f_i^U(K)-(1+r)d_{i-1}^U+d_i^U \quad \forall i\geq 3 \qquad (\varphi_i) \qquad (3)$$

$$c_1^C = f_1^C(K)+\epsilon(b)-(m+r)(b+R)+(1+r^*)R \qquad (\Psi_1) \qquad (4)$$
$$c_2^C = f_2^C(K)-(1+r)\epsilon(b)+d_2^C-(1-m)(1+r)(b+R) \qquad (\Psi_2) \qquad (5)$$
$$c_i^C = f_i^C(K)-(1+r)d_{i-1}^C+d_i^C \qquad (\Psi_3) \qquad (6)$$

The utility function takes the standard CRRA form, $u(c)=\frac{c^{1-\rho}}{1-\rho}$, and we also assume $\beta = 1/(1+r)$. This ensures that, absent any crisis, the optimal level of consumption would be constant across time. In turn, this implies that consumption is simply equal to the annuity of inter-temporal income, net of inherited debt and cost of reserves:

$$\forall i\geq 1, \quad c_i^U = r\left(\sum_{t\geq 1}\frac{f_t^U(K)}{(1+r)^t}-b(1+r)-R(r-r^*)\right) \qquad (7)$$

The simplifying assumption[4] $K\approx b$ ensures the model can be summarized in terms of two relatively simple Euler equations. This assumption will be relaxed in the recursive dynamic model of section 5.

4.2 Euler Equation for Capital/Debt

The Euler equation for the stock of capital is (subscripts denote partial derivatives)

$$\sum_{i\geq 1}\varphi_i f_K^U(K)-\varphi_1((m+r)+(1-m))+\Psi_1[f_{K,1}^C(K)-(m+r)+\epsilon_b(K)] \qquad (8)$$
$$+\Psi_2[f_{K,2}^C(K)-\epsilon_b(K)(1+r)+(1-m)(1+r)]+\sum_{i\geq 3}\Psi_i f_K^C(K)=0$$

We define two variables x and y that summarize the depth of the balance of payment crisis: $0<x=\frac{c_2^C-c_1^C}{c_1^C}<<1$ and $0<y=\frac{c_1^U-c_1^C}{c_1^C}<<1$. x is the loss in consumption compared to the state of nature without crisis, whereas y is the growth rate of consumption in the recovery

[4]The exact relationship is $K=b-R$, but the approximation is made without loss of generality, given that R is small.

phase, immediately after the crisis. The ratios of marginal utilities can then be expressed simply in terms of x, y and the preference parameters

$$\frac{\Psi_1}{\varphi_1} = \frac{\pi}{1-\pi}\left(\frac{c_1^U}{c_1^C}\right)^{\rho} \approx \frac{\pi}{1-\pi}(1+\rho y) \tag{9}$$

$$\frac{\Psi_2}{\Psi_1} = \beta\left(\frac{c_1^C}{c_2^C}\right)^{\rho} \approx \beta(1-\rho x) = \beta(1-x/\sigma) \tag{10}$$

Using the First-Order Conditions $\varphi_i = (1+r)\varphi_{i+1}, \forall i \geq 1$ and $\Psi_i = (1+r)\Psi_{i+1}, \forall i \geq 3$, the Euler equation can thus be re-written as

$$\begin{aligned}(1-\pi)(c_1^U)^{-\rho}\left[\sum_{t\geq 1}\frac{f_K^U(K)}{(1+r)^{t-1}} - (1+r)\right] + \beta\pi(1-x/\sigma)(1+\rho y))(c_1^U)^{-\rho}\left[\sum_{t\geq 3}\frac{f_K^C(K)}{(1+r)^{t-1}}\right] \\ + \pi(1+\rho y)(c_1^U)^{-\rho}\left[f_{K,1}^C(K) - (m+r) + \epsilon_b(K)\right] \\ + \pi\beta(1-x/\sigma)(1+\rho y)(c_1^U)^{-\rho}\left[f_{K,2}^C(K) - \epsilon_b(K)(1+r) - (1-m)(1+r)\right] = 0\end{aligned} \tag{11}$$

After some algebra, the Euler equation becomes

$$\mathbb{E}\left[\sum_{t\geq 1}\frac{f_{K,t}^U(K)}{(1+r)^{t-1}}\right] \approx 1 + r + \pi\frac{x}{\sigma}(1+\rho y)(m - \epsilon_b(K)) \tag{12}$$

Or equivalently, the formula can be expressed using the internal rate of return for the marginal investment project:

$$\mathbb{E}\left[\sum_{t\geq 1}\frac{f_{K,t}^U(K)}{(1+r+\theta)(1+r)^{t-1}}\right] - 1 = 0 \tag{13}$$

where $\theta = \pi\frac{x}{\sigma}(1+\rho y)(m-\epsilon_b(K))$ is a risk premium due to the utility costs of external shocks.

Since the marginal product of capital is decreasing in K, the optimal level of capital (and debt, in this simplified model) is:

- decreasing in π
- increasing in the intertemporal elasticity of substitution σ
- decreasing in risk-aversion ρ
- decreasing in $\epsilon_b(K)$, i.e. the sensitivity of the external shock to debt (which equals the capital stock in this simplified set up)
- decreasing in the ratio of short-term debt to total debt m

Note also that it is crucial, for a risk-premium to exist, that the intertemporal elasticity of substitution (IES) be different from 0: when the IES is 0, there is no cost of being temporary unable to borrow. In addition, risk aversion magnifies the utility cost of the borrowing constraint in the bad state of nature. Note however that even a risk-neutral agent would take into account the utility costs of its inability to borrow, because the expected utility is affected by the liquidity constraint. However it is essential that the IES be different from 0, otherwise there is no utility cost of the liquidity constraint. A For illustration purposes only, a realistic calibration of the model suggests that the risk-premium θ is sizeable, and could amount to 2-3 percent.[5]

The risk-premium only applies to the first period, since this is the one where the shock occurs, but the formula is easily extended in a model where the crisis can occurring at any future point in time. Appendix 2 shows that in such a model, the Euler equation is:

$$\mathbb{E}_0 \left[\sum_{t \geq 0} \frac{\tilde{f}_{K,t}(K) - \tilde{m}_t^s}{1 + \tilde{\theta}_t^s} \right] = 0 \tag{14}$$

where $\tilde{m_t}$ is a risk adjusted debt service and $\tilde{\theta_t}$ is the risk adjusted discount rate (see Appendix 2 for the precise definitions).

4.3 Euler Equation for Reserves

The optimal level of reserves has typically been discussed in the context of simple rules, for instance, reserves should cover short-term debt (the Greenspan-Guidotti rule), imports, etc. Jeanne and Ranciere (2011) more clearly showed that reserves equal the share of short-term debt and the reduction in output growth to achieve full-insurance. Such results can be derived from the budget constraint in the first period of the crisis:

$$R = \frac{c_1^C - f_1^C(K)}{1 - m - (r - r^*)} - \frac{\epsilon(K)}{1 - m - (r - r^*)} + \frac{m + r}{1 - m(r - r)^*} b \tag{15}$$

It is thus a general result that reserves will cover the financing gap created by the fall in output and the amortization of short-term debt (that is not being rolled-over). An additional issue, however, is to determine the level of consumption during the crisis and, importantly, the level of debt that should be taken given the risk that debt is not rolled-over. These questions need to be answered using the Euler equation for capital (and thus debt) and the Euler equation for international reserves, which is:

$$\underbrace{(1 - (r - r^*) - m)\Psi_1}_{\text{Benefit in crisis period}} = \underbrace{(1 - m)(1 + r)\Psi_2}_{\text{cost of debt}} + \underbrace{(r - r^*)\varphi_1}_{\text{opportunity cost of reserves}} \tag{16}$$

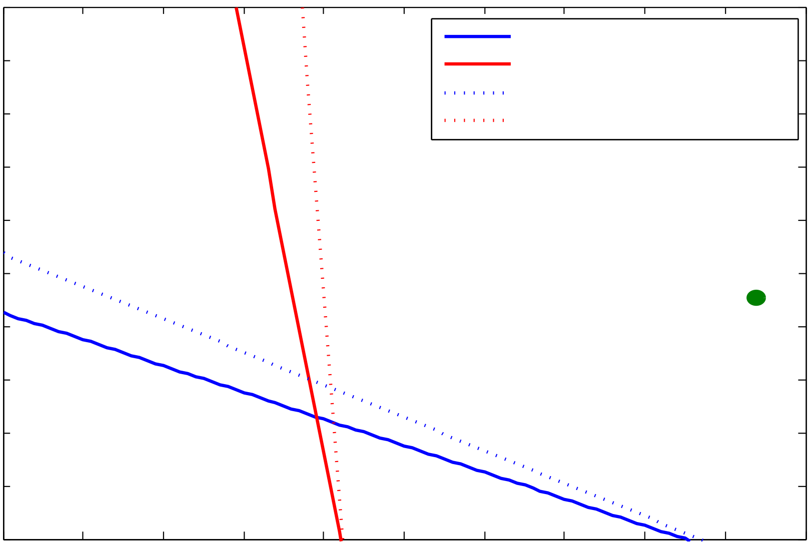
The term in the left-hand side of the Euler equation is the benefit (in terms of marginal utility) of reserves in the crisis period. The second term is the cost, in period 2 of the normal state of nature, of the long-term liabilities that financed reserves. The third term is the opportunity cost of reserves, which is 'paid' in the good state.

[5] The calibration is $\pi = 0.1$, $x = y = 0.05$, $\sigma = 0.5$, $\rho = 2$, $r = 0.05$, $f(K) = K^{1/3}$, $K^* = 0.46$, $Y^* = 0.77$, $f'(K*) = 0.55$, $m = 0.2$ and $\epsilon_b(b) = -2$.

4.4 System of Euler Equations

The Euler equation for the accumulation of reserves is linear, decreasing in the ratios of marginal utilities φ_1/Ψ_1 and Ψ_2/Ψ_1 and can thus be represented as the blue locus in Figure 2, where the horizontal axis is the ratio of marginal utilities across periods (in the crisis state), and the vertical axis is the ratio of marginal utilities across states of nature, weighted by their respective probabilities. The intersection of the locus with the horizontal axis can be solved by setting $\varphi_1 = 0$ in equation 2, and similarly, the intersection with the vertical axis is solved by setting $\Psi_2 = 0$. As a first approximation, the Euler equation for capital/debt can also be represented by a downward sloping linear locus.[6] The interior equilibrium is at the intersection of the two loci. Utility is higher as the equilibrium point moves to the top-right corner; in particular, the 'bliss' point, characterizing full insurance (i.e. even consumption during the crisis and post-crisis period, and equal utility across states of nature), is represented by the green circle, and the distance between the bliss point and the equilibrium captures the utility losses in both dimensions. Some comparative statics can be discussed simply in this framework. For instance, when the yield on reserves rises, the locus of the reserves' Euler equation moves to the right and becomes steeper, bringing the equilibrium closer to the bliss point (the debt Euler equation locus is unchanged). This allows for a higher stock of capital, which also tilts the debt/capital Euler equation towards the right. The comparative statics is unambiguous in showing higher consumption smoothing along both dimensions (i.e. consumption smoothing in the crisis state as well as reduced risk across states of nature).

Figure 2: System of Euler Equations



[6] This assumes that $f_K(K)$ is relatively insensitive to K. Equation 8 is equivalent to the equation: $\varphi_1 \left(f_K^U(K)\frac{1+r}{r} - (1+r)\right) + \Psi_1 \left(f_{K,1}^C - (m+r) + \epsilon_b(K)\right) + \Psi_2 \left(f_{K,2}^C(1+1/r) - \epsilon_b(K)(1+r) + (1-m)(1+r)\right) = 0$

5 Infinite-Horizon Model

5.1 Model

We now extend the model to a dynamic, rational expectation, recursive model where the country is allowed to choose the stocks of debt, reserves and capital. Defining a recursive formulation is more complex than in standard open economy models because the choices of debt and reserves made at a period t affect the financing possibilities between period $t+1$ and $t+2$. Two modeling options are possible in this situation. One option, which is computationally intensive, is to track the stocks of debt and reserves over two consecutive periods, thus doubling the number of state variables. The second option, that is chosen in this paper, is to define as one 'period' t two consecutive periods $t,1$ and $t,2$. This modeling choice has also the advantage to follow naturally the three-period model presented earlier. The optimization problem involves two value functions, for (i) the normal state and (ii) the crisis state, which we describe now.

Normal state

The problem in the normal state (or *unconstrained* state, note with superscript U) is to maximize current utility, plus the discounted expected value function, which is a weighted average of the value function in the normal state (V^U, with weight $(1-\pi)$) and of the value function in the crisis state (V^C, with weight π). The value function is a function of the three endogenous states b, R and k:

$$V_t^U(b_{t-1}, R_{t-1}, K_{t-1}) = \max_{b_t, R_t, i_t} u(c_t^U) + \beta \left\{(1-\pi)V_{t+1}^U(b_t, R_t, K_t) + \pi V_t^C(b_t, R_t, K_t)\right\} \tag{17}$$

where consumption in a normal state is

$$c_t^U = f^U(K_{t-1}) + b_t - (1+r)b_{t-1} - (R_t - R_{t-1}) - i_t - \frac{\chi}{2}\left(\frac{i_t - \delta K_{t-1}}{K_{t-1}}\right)^2$$

and investment is $i_t = K_t - (1-\delta)K_{t-1}$. Consumption in the normal state is thus equal to output, plus net financing, minus reserve accumulation, minus investment, and minus capital adjustment costs (the last term, which is defined such that capital adjustment costs are 0 in steady-state).

Crisis state

The maximization problem in the crisis state (or *constrained* state, note with superscript C) is constructed in a similar way, taking now into account the unavailability of financing in the crisis sub-period $t,1$:

$$\begin{aligned} V_t^C(b_{t-1}, R_{t-1}, K_{t-1}) = \max_{b_t, R_t, i_t} & \left\{u(c_{t,1}^C) + \beta^{1/2}u(c_{t,2}^C)\right\} \\ & + \beta\left\{(1-\pi)V_{t+1}^U(b_t, R_t, K_t) + \pi V_{t+1}^C(b_t, R_t, K_t)\right\} \end{aligned} \tag{18}$$

where the consumption levels in the two sub-periods (crisis and post-crisis) are defined as:

$$c^C_{t,1} = f^C_1(K_{t-1}) + R_{t-1} - i_t - \frac{\chi}{2}\left(\frac{i_t - \delta K_{t-1}}{K_{t-1}}\right)^2 \tag{19}$$

$$c^C_{t,2} = f^C_2(K_{t-1}) + b_t - (1+r)b_{t-1} - R_t - i_t - \frac{\chi}{2}\left(\frac{i_t - \delta K_{t-1}}{K_{t-1}}\right)^2 \tag{20}$$

In the first period of the crisis, consumption is constrained to output and to accumulated reserves. All reserves are consumed in the crisis period because of two simplifying assumptions: (i) the country will be able to borrow immediately after the crisis period is over (or equivalently, the crisis only lasts one sub-period) ; (ii) the extent of the crisis, if it is to happen, is known ex-ante. In the period following the crisis, the economy recovers its access to finance and the budget constraint is similar to that in the normal state.

The derivations of the first-order conditions and the envelope conditions are presented in Appendix 2. The Euler equations are akin to equations for portfolio choices, except that the marginal utilities involved in the crisis state depend on whether the asset (debt, reserves, or capital) pays off in the crisis sub-period $(t+1,1)$ or in post-crisis recovery $(t+1,2)$. In addition, there are two inequality constraints: $R_t \geq 0$ and $i_t \geq 0 \Longleftrightarrow K_t \geq K_t(1-\delta)$.
For the unconstrained state, the Euler equations are:

$$Eq/b_t: \quad u_c(c^U_t) = \beta\left\{(1-\pi)(1+r)u_c(c^U_{t+1}) + \pi(1+r)\beta^{1/2}u_c(c^C_{t+1,2})\right\} = 0 \tag{21}$$

$$Eq/R_t: \quad u_c(c^U_t) = \beta\left\{(1-\pi)u_c(c^U_{t+1}) + \pi u_c(c^C_{t+1,1})\right\} = 0 \tag{22}$$

$$\begin{aligned} Eq/K_t: \quad & u_c(c^U_t)(1+\chi(K_t - K_{t-1})) = \\ & \beta(1-\pi)u_c(c^U_{t+1})\left[f^U_K(K_t) + 1 - \delta - \chi\frac{K_{t+1}}{K_t^2}\left(\frac{K_{t+1}}{K_t} - 1\right)\right] + \\ & \beta\pi u_c(c^C_{t+1,1})\left[f^C_{K,1}(K_t) + 1 - \delta - \chi\frac{K_{t+1}}{K_t^2}\left(\frac{K_{t+1}}{K_t} - 1\right)\right] + \\ & \beta\pi u_c(c^C_{t+1,2})\left[f^C_{K,2}(K_t) + 1 - \delta - \chi\frac{K_{t+1}}{K_t^2}\left(\frac{K_{t+1}}{K_t} - 1\right)\right] \end{aligned} \tag{23}$$

The equations are nearly identical for the constrained state, except that the left-hand side is the marginal utility of the second sub-period (i.e. $\beta^{1/2}u_c(c^C_{t,2})$)

5.2 Numerical Application

In its most general formulation, the computational problem is difficult, since one needs to solve a dynamic, recursive, rational exceptional equilibrium with 3 endogenous state variables (k, b, R) and in presence of two inequality constraints ($R \geq 0$ and $i \geq 0$). In addition, there is one exogenous state, which affects the type of intertemporal problem faced by the country: if the country suffers from a sudden stop, external financing is exogenous; if there is no crisis, the country can borrow any desired amount at a fixed interest rate. Fortunately, the setup can be modified, without any meaningful change, to simplify the problem greatly. The current 'period' can be defined as a sequence of non-crisis period, followed by the possibility that a crisis occurs (i.e. in reverse order from the more natural order presented above). In that setup, only the net asset position matters and the two endogenous states become the net asset position n and capital k. In addition, the maximization problem for the current period already takes into account uncertainty and one needs not add the state of the economy as an exogenous state.

Thus, the simplified model collapses to a two-state dynamic optimization problem, which can be solved accurately using value function. The inequality constraints are taken into account for the bounds given to the search algorithm. The state-space is discretized over a grid made of 15*15 grid points.

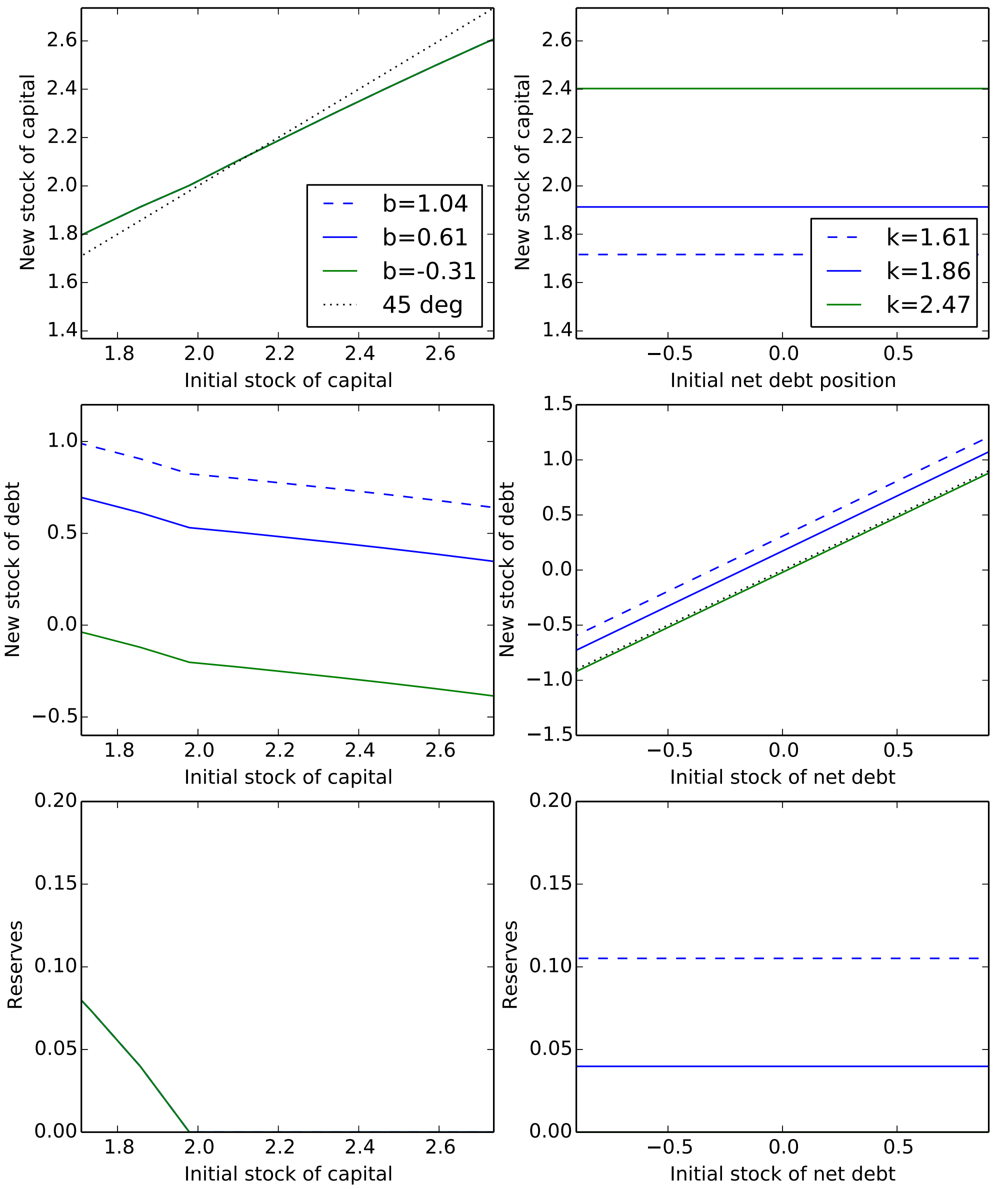
The model functional forms and parameters are standard. Utility is of the CRRA type, with risk aversion set to 4. The production function is $f_j^s(K) = A_j^s K^\alpha$, where $s \in \{U; C\}$ and $j \in \{t; t,1; t,2\}$. The parameters are presented in Table 3.

Table 3: Parameters in the baseline calibration

Parameters	value	description
α	0.33	Share of capital income in total income
δ	0.1	Depreciation rate of public and private capital
β	0.95	Utility discount rate
r	$1/\beta$	Real interest rate on debt set at 4 percent
r^*	0	Real interest rate on reserves set at 0
ρ	4.0	Coefficient of Relative Risk Aversion
A^U	1.0	TFP in normal state
$A_{C,1}$	0.9	TFP in crisis period
$A_{C,2}$	1.0	TFP post-crisis
$\epsilon_b(b)$	0	all debt is rolled-over

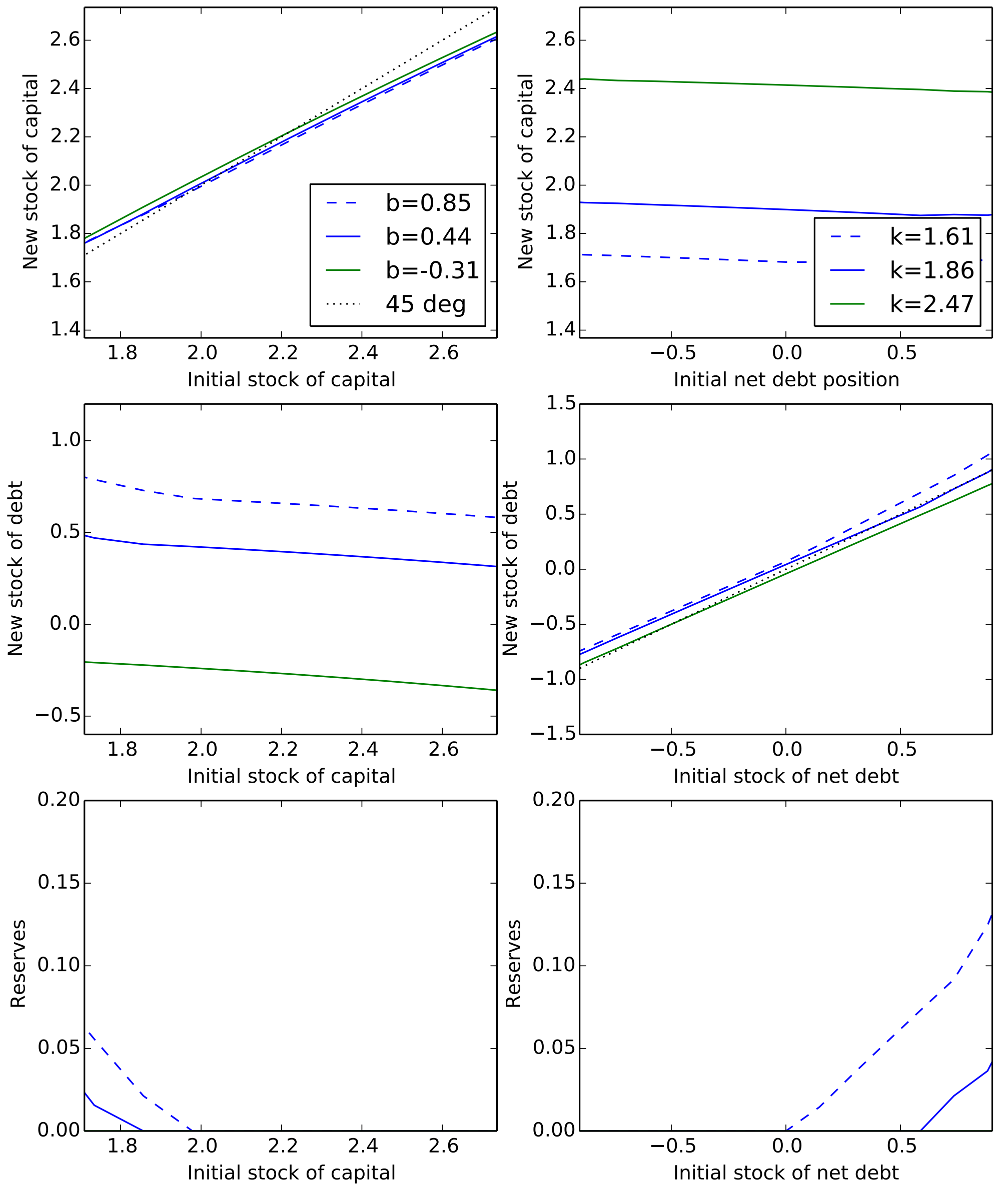
The baseline results are presented in Figure 3. All variables are expressed in proportion to output in the (safe) steady-state, which is equal to 1.46. The decision rule for the stock of capital, in the first row of Figure 3, is a standard rule for an open-economy with capital adjustment costs. The steady-state stock of capital is 2.12, implying that with the baseline calibration the possibility that a crisis occurs reduces the optimal stock of capital by around 2 percent (see below for further comparative statics at the steady-state). The decision rule for the stock of capital is independent from the initial net asset position in this version of the model where all debt is rolled-over (i.e. $\epsilon_b = 0$).

Figure 3: Policy functions, with $\epsilon(b) = 0$ (in percent of steady-state output of normal state)



Gross borrowing is decreasing in the stock of capital, and an increasing function of initial debt, because of roll-over needs (second row). Gross borrowing is a piece-wise linear function of the initial stock of capital because for low levels of the stock of capital, it is worth accumulating reserves, whereas for levels of the capital stock higher than 1.98, the welfare losses due to the sudden stop are not large enough to justify reserve accumulation.

Figure 4: Policy functions, with $\epsilon(b) = -0.15b$ (in percent of steady-state output of normal state)



Again, in this model, the accumulation of reserves is independent from the stock of debt because the sudden stop is exogenous. Figure 4 shows the same policy functions under the assumption that lenders do not roll-over 15 percent of external debt in the sudden stop ($i.e. \epsilon(b) = -0.15b$). In this version of the model, the decision rule for the stock of capital is a decreasing function of the initial debt position (top row charts), and higher inherited debt triggers a larger accumulation of reserves (last row).

Figures 5 and 6 present a few comparative statics, computed at the steady-state stock of capital. The optimal steady-state stock of capital is a decreasing function of the extent of crisis (captured by the TFP parameter $A_{c,2}$, and is a decreasing, convex function of the probability of the crisis. For a crisis probability of 15 percent, the steady-state stock of capital stock is reduced by 2.5 to 5 percent. As a result, the (implicit) expected rate of return for investment is a non-monotonous function of the crisis probability π: on one hand, expected TFP is decreasing in π ; on the other hand, the marginal product of capital is decreasing with K, and K is itself decreasing in π.

Reserves are accumulated if the welfare gains exceed the opportunity costs: depending on the depth of the crisis, this happens for crisis probabilities ranging between 3 to 13 percent. The effect on the depth of the consumption drop is shown in the bottom-right chart of Figure 5. At low levels of π, reserves are not accumulated and thus the consumption drop is given. At higher levels of π, reserves are accumulated, reducing the fall in consumption during the crisis. Finally, it is worthwhile noting that the optimal level of reserves is fairly sensitive to both risk aversion and the yield on reserves (Figure 6).

6 Concluding remarks

The literature on the balance sheets of countries and governments has mostly followed the Eaton-Gersovitz model, linking the accumulation and the price of external debt to the probability of default. But default and debt restructuring events are relative uncommon, and not all external financing decisions should thus be viewed in light of this model. In this paper, we present a model of debt and central bank reserves that focuses on *exogenous shocks* to the balance of payments. Although holding liquid foreign assets is costly for central banks, as this decision implies foregoing income from high yield assets, the strategy may be worthwhile when the costs of a balance of payment crisis are high enough.

The modeling strategy is centered explicitly on the inability to borrow in periods of sudden stops. This risk affects the government's optimal investment levels as well as the financing and central bank reserves decisions. The model thus solves a portfolio decision involving three asset classes: external debt, central bank reserves, and physical capital. The three assets are imperfect substitutes because of different returns, different 'liquidity' characteristics and restrictions on whether the asset position is negative or positive: (i) external debt is a safe asset with a high rate of return, but the country can only be a debtor in that asset (ii) physical capital is a risky asset, and it cannot be liquidated during sudden stops; (iii) central bank reserves do not yield any income (and the country cannot borrow using this asset), but they pay off in the sudden stop.

Figure 5: Comparitive Statics for π and A_c (in percent of steady-state output of normal state

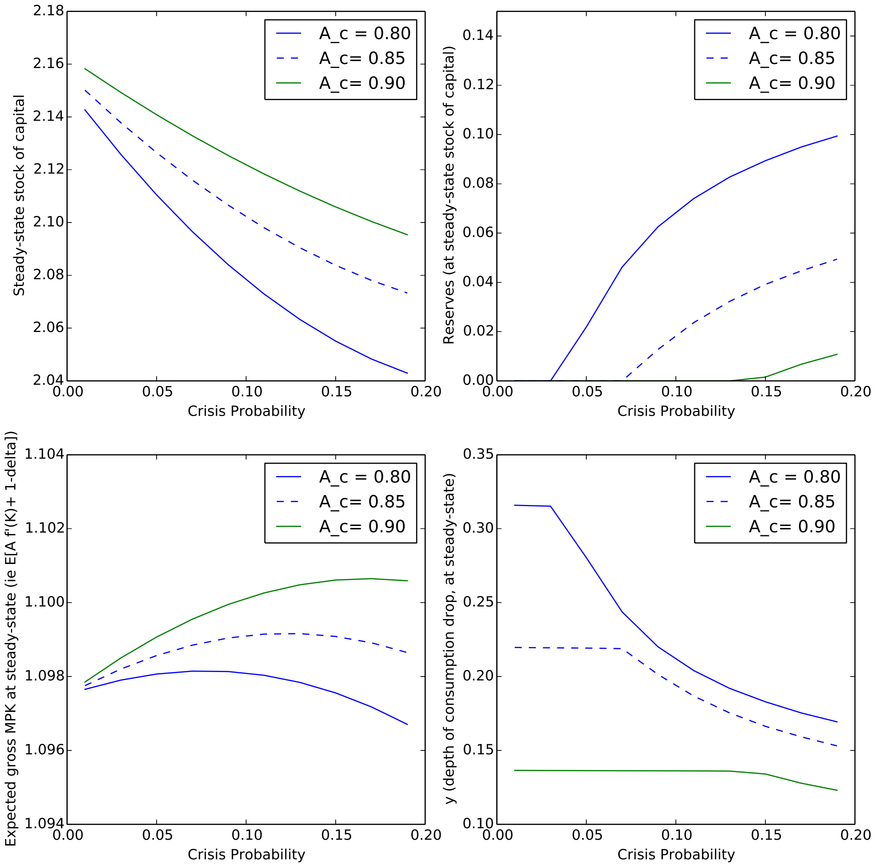
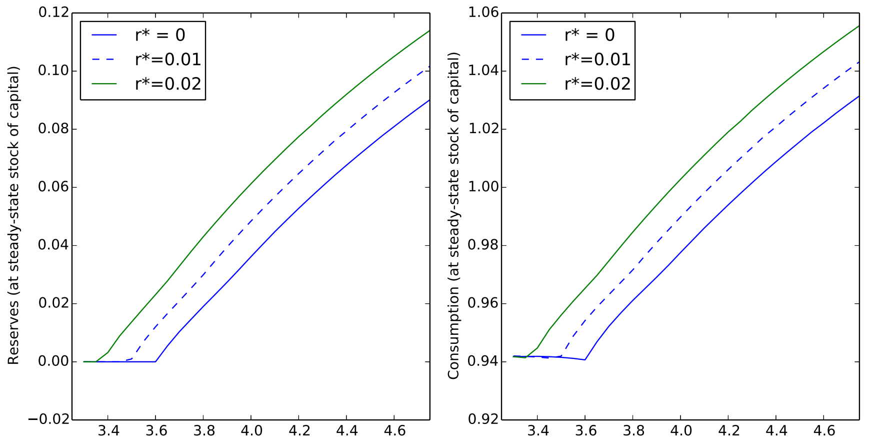


Figure 6: Comparitive Statics for ρ and $r*$ (in percent of steady-state output of normal state



The full model involves three endogenous state variables, two regimes, and two inequality constraints, but a simple version of the model is solved recursively under the assumption that crisis periods are immediately followed by a recovery, i.e. sudden stops only last one period. The decision rules and comparative statics are intuitive, and provide a first quantitative application of the model. Future research would usefully focus on solving the full model under a more complex stochastic structure.

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Appendix 1

Table 4: Description of data

Variable	Data sources
Reserves, in US$	World Bank
GDP, in US$	IMF World Econmic Outlook Database
Imports of goods and services	IMF World Econmic Outlook Database
Terms of trade	IMF World Econmic Outlook Database
FDI net inflow, in US$	IMF World Econmic Outlook Database
Official transfers (current account), in US$	IMF World Econmic Outlook Database
Offical financing (financial account), in US$	IMF World Econmic Outlook Database
Disasters (number of affected people)	EMDAT database
Population	IMF World Econmic Outlook Database
Sovereign debt restructuring (private creditors)	Cruces and Trebesch (2013)
Sovereign debt restructuring (Paris Club)	Das, Papaioannou and Trebesch (2011)
Currency crisis, debt crisis (domestic and external)	Reinhard and Rogoff (2010)
Banking crisis	Reinhard and Rogoff (2010)

Country and period coverage

Afghanistan, I.R. of (2008-2012); Albania (1993-2012); Algeria (1969-2012); Angola (1995-2012); Antigua and Barbuda (1975-2012); Argentina (1988-2012); Armenia (1992-2012); Azerbaijan, Rep. of (1993-2012); Bahamas, The (1989-2012); Bahrain, Kingdom of (1968-2012); Bangladesh (1972-2012); Barbados (1971-2012); Belarus (1994-2012); Belize (1976-2012); Benin (1970-2012); Bhutan (1983-2012); Bolivia (1985-2012); Bosnia-Herzegovina (1998-2012); Botswana (1976-2012); Brazil (1992-2012); Brunei Darussalam (1999-2012); Bulgaria (1991-2012); Burkina Faso (1969-2012); Burundi (1980-2012); Cambodia (1993-2012); Cameroon (1980-2012); Cape Verde (1976-2012); Central African Rep. (1969-2012); Chad (1969-2012); Chile (1968-2012); China Mainland (1977-2012); Colombia (1969-2012); Comoros (1980-2012); Congo, Republic of (1969-2012); Costa Rica (1967-2012); Croatia (1992-2012); Cote d'Ivoire (1968-2012); Djibouti (1990-2012); Dominica (1975-2012); Dominican Republic (1969-2012); Ecuador (1979-2012); Egypt (1969-2012); El Salvador (1981-2012); Equatorial Guinea (1982-2012); Eritrea (1995-2011); Estonia (1993-2012); Ethiopia (1967-2009); Fiji (1969-2012); Gabon (1969-2012); Gambia, The (1980-2012); Georgia (1996-2012); Ghana (1986-2012); Grenada (1973-2012); Guatemala (1967-2012); Guinea (1991-2011); Guinea-Bissau (1986-2012); Guyana (1969-2012); Haiti (1969-2012); Honduras (1969-2012); Hungary (1983-2012); India (1969-2012); Indonesia (1969-2012); Iran, I.R. of (1969-1982); Iraq (2005-2012); Jamaica (1967-2012); Jordan (1967-2012); Kazakhstan (1994-2012); Kenya (1967-2012); Kuwait (1969-2012); Kyrgyz Republic (1993-2012); Latvia (1993-2012); Lebanon (1990-2012); Lesotho (1980-2006); Libya (1969-2010); Lithuania (1995-2012); Macedonia, FYR (1993-2012); Madagascar (1980-2012); Malawi (1969-2012); Malaysia (1969-2012); Maldives (1976-2012); Mali (1980-2012); Mauritania (1969-2012); Mauritius (1969-2012); Mexico (1969-2012); Moldova (1992-2012); Mongolia (1992-2012); Morocco (1969-2012); Mozambique (1984-2012); Myanmar (1969-2012); Namibia (1992-2012); Nepal (1969-2012); Nicaragua (1990-2012); Niger (1969-2012); Nigeria (1969-2012); Oman (1970-2012); Pakistan (1969-2012); Panama (1969-2012); Papua New Guinea (1973-2012); Paraguay (1969-2012); Peru (1989-2012); Philippines (1969-2012); Poland (1979-2012); Qatar (1990-2012); Romania (1975-2012); Russian Federation (1993-2012); Rwanda (1969-2012); Saudi Arabia (1976-2012); Senegal (1969-2012); Serbia, Republic of (1999-2012); Seychelles (1976-2012); Sierra Leone (1969-2012); Solomon Islands (1980-2012); South Africa (1967-2012); Sri Lanka (1969-2012); St. Kitts and Nevis (1981-2012); St. Lucia (1975-2012); St. Vincent and Grens. (1976-2012); Sudan (1989-2012); Suriname (1994-2012); Swaziland (1974-2012); Syrian

Arab Republic (1969-2010); Tajikistan (1997-2012); Tanzania (1969-2012); Thailand (1969-2012); Timor-Leste (2002-2012); Togo (1969-2012); Trinidad and Tobago (1969-2012); Tunisia (1969-2012); Turkey (1985-2012); Turkmenistan (1996-1999); Uganda (1977-2012); Ukraine (1992-2012); United Arab Emirates (1973-2012); Uruguay (1978-2012); Venezuela, Rep. Bol. (1968-2012); Vietnam (1995-2012); Yemen, Republic of (1990-2012); Zambia (1969-2012)

Table 5: Logit models, by groups

Variables	(1) logit	(2) logit by ExR regime	(3) logit by ExR regime	(4) logit by ExR regime	(5) logit by ExR regime	(6) logit LIC/non-LIC	(7) logit LIC/non-LIC	(8) logit LIC/non-LIC	(9) logit LIC/non-LIC
Balance of Payt shock	0.357***	0.452***	0.357***	0.352***	0.357***	0.302**	0.359***	0.363***	0.363***
	[3.350]	[3.083]	[3.351]	[3.298]	[3.347]	[2.112]	[3.367]	[3.405]	[3.401]
Natutral disaster	-0.0513	-0.0501	-0.0124	-0.0494	-0.0510	-0.0511	-0.137	-0.0543	-0.0486
	[-0.311]	[-0.305]	[-0.0587]	[-0.300]	[-0.309]	[-0.310]	[-0.593]	[-0.330]	[-0.295]
Debt rest. (with priv. creditors)	-0.0485	-0.0371	-0.0490	-0.242	-0.0478	-0.0561	-0.0489	0.238	-0.0340
	[-0.167]	[-0.127]	[-0.169]	[-0.693]	[-0.164]	[-0.194]	[-0.169]	[0.707]	[-0.117]
Debt rest. (with Paris Club)	-0.571***	-0.573***	-0.571***	-0.577***	-0.600**	-0.557***	-0.559***	-0.559***	-0.779**
	[-2.710]	[-2.720]	[-2.711]	[-2.743]	[-2.266]	[-2.632]	[-2.645]	[-2.642]	[-2.047]
I(fixed ExR)	-0.429***	-0.340**	-0.418***	-0.449***	-0.434***	-0.422***	-0.424***	-0.421***	-0.425***
	[-4.026]	[-2.390]	[-3.712]	[-4.177]	[-3.959]	[-3.947]	[-3.969]	[-3.932]	[-3.979]
LIC						-0.112	-0.0739	-0.0241	-0.0727
						[-0.774]	[-0.650]	[-0.222]	[-0.656]
Balance of Payt shock * I(fixed ExR)		-0.200							
		[-0.942]							
Natutral disaster * I(fixed ExR)			-0.0968						
			[-0.286]						
Debt rest.(with priv. creditors) * I(fixed ExR)				0.694					
				[1.136]					
Debt rest.(with Paris Club) * I(fixed ExR)					0.0810				
					[0.188]				
Balance of Payt shock * I(LIC)						0.131			
						[0.612]			
Natutral disaster * I(LIC)							0.183		
							[0.558]		
Debt rest.(with priv. creditors) * I(LIC)								-0.978	
								[-1.410]	
Debt rest.(with Paris Club) * I(LIC)									0.328
									[0.721]
Constant	-1.659***	-1.702***	-1.664***	-1.647***	-1.656***	-1.618***	-1.631***	-1.655***	-1.633***
	[-18.20]	[-16.32]	[-17.90]	[-18.12]	[-18.01]	[-15.42]	[-16.34]	[-16.49]	[-16.45]
Observations	2,971	2,971	2,971	2,971	2,971	2,971	2,971	2,971	2,971

Robust z-statistics in brackets
*** $p < 0.01$, ** $p < 0.05$, *$p < 0.1$

Table 6: Logit and Probit models - Alternative identification of balance of payt. shocks

Variables	(1) logit pooled	(2) probit pooled	(3) logit fixed effects (FE)	(4) logit fixed effects (FE)	(5) logit fixed effects (FE)	(6) logit fixed effects (FE)
Terms of trade shock	0.568*** [6.427]	0.336*** [6.340]	0.585*** [6.530]	0.628*** [7.027]	0.669*** [4.393]	0.643*** [4.283]
Terms of trade shock (t-1)	0.122 [1.297]	0.0719 [1.303]	0.150 [1.573]			
FDI shock	-0.127 [-1.190]	-0.0743 [-1.209]	-0.117 [-1.062]	-0.0676 [-0.617]	-0.0379 [-0.183]	-0.102 [-0.506]
FDI shock (t-1)	-0.187* [-1.715]	-0.108* [-1.730]	-0.173 [-1.556]			
Official financing shock	0.141 [1.371]	0.0818 [1.363]	0.176* [1.663]	0.212** [2.017]	0.0923 [0.433]	0.0986 [0.472]
Official financing shock (t-1)	-0.00798 [-0.0764]	-0.00843 [-0.139]	0.0278 [0.256]			
Natutral disaster	-0.105 [-0.939]	-0.0608 [-0.939]	-0.110 [-0.886]	-0.0935 [-0.761]	-0.411* [-1.827]	-0.364* [-1.696]
Natutral disaster (t-1)	-0.00445 [-0.0402]	-0.00499 [-0.0773]	-0.00951 [-0.0780]			
Debt restructuring (with priv. creditors)	-0.156 [-0.668]	-0.0882 [-0.666]	-0.199 [-0.842]	-0.182 [-0.778]	0.0323 [0.0990]	-0.000227 [-0.000717]
Debt restructuring (with priv. creditors) (t-1)	-0.0320 [-0.145]	-0.0220 [-0.173]	-0.0751 [-0.331]			
Debt restructuring (with Paris Club)	-0.366** [-2.457]	-0.205** [-2.446]	-0.342** [-2.237]	-0.320** [-2.111]	-0.390 [-1.466]	-0.280 [-1.094]
Debt restructuring (with Paris Club) (t-1)	-0.0757 [-0.549]	-0.0442 [-0.553]	-0.0639 [-0.441]			
Currency crisis					0.727*** [4.574]	
Domestic debt crisis					0.127 [0.335]	
External debt crisis					-0.0912 [-0.463]	
Banking crisis					0.207 [1.239]	
Currency crisis (t-1)						-0.268 [-1.577]
Domestic debt crisis (t-1)						0.414 [1.101]
External debt crisis (t-1)						0.116 [0.631]
Banking crisis (t-1)						-0.159 [-0.901]
Constant	-1.279*** [-22.62]	-0.779*** [-23.87]				
Observations	4,479	4,479	4,476	4,613	1,604	1,604
Number of countries			137	137	44	44

Robust z-statistics in brackets
*** $p < 0.01$, ** $p < 0.05$, *$p < 0.1$

Appendix 2

Derivations of the three-period model

The First Order Conditions are

$$\begin{aligned}
FOC/c_i^U: &\quad (1-\pi)\beta^{i-1}u_c(c_t^U)=\varphi_i \quad ,\forall i\geq 1 && (24)\\
FOC/d_i^U: &\quad \varphi_i=(1+r)\varphi_{i+1} \quad ,\forall i\geq 1 && (25)\\
FOC/c_1^C: &\quad \pi u_c(c_1^C)=\Psi_1 && (26)\\
FOC/c_2^C: &\quad \pi(\beta u_c(c_2^C)=\Psi_2 && (27)\\
FOC/c_i^C: &\quad \pi\beta u_c(c_2^C)=\Psi_i \quad ,\forall i\geq 3 && (28)\\
FOC/d_i^C: &\quad \Psi_i=(1+r)\Psi_{i+1}\Longrightarrow \beta^{i-1}u_c(c_i^C)=(1+r)\beta^i u_c(c_{i+1}^C)\Longrightarrow c_i^C=c_{i+1}^C \quad ,\forall i\geq 2 && (29)\\
FOC/d_i^K: &\quad 0=\sum_i \varphi_i f_K^U(K)-\varphi_1(m+r+(1-m)- && (30)\\
&\quad \sum_i \Psi_i f_{K,1}^C(K)+(m+r-\epsilon_b(b))\Psi_1\\
&\quad +(1-m)(1+r)\varphi_2+(1+r)(1+1-m)\epsilon_b(b)\Psi_2
\end{aligned}$$

Extension of the three-period model

The maximization problem can be written as follows (the lagrangian multipliers are shown in brackets for their respective budget constraints):

$$\max\sum_{t=0}^{\infty}\beta^t(1-\pi)^t\left[u(c_t^{u^t})+\pi\left(\beta u(c_{t+1}^{u^t d})+\sum_{s=2}^{\infty}\beta^s u(c_{t+s}^{u^t d})\right)\right] \tag{31}$$

s.t., $\forall t\geq 0$

$$\begin{cases}
c_t^{u^t}=f_t(D)-rD-(1+r)d_{t-1}^{u^{t-1}}+d_t^{u^t} & (\varphi_t)\\
c_{t+1}^{u^t d}=f_{t+1}(D)-rD-(1+r)d_t^{u^t}+\epsilon_{t+1}(D) & (\Psi_t)\\
c_{t+2}^{u^t d}=f_{t+2}(D)-rD-(1+r)\epsilon_{t+1}(D)+d_{t+2}^{u^t} & (\chi_t)\\
c_{t+s}^{u^t d}=f_{t+s}(D)-rD-(1+r)d_{t+s-1}^{u^t}+d_{t+s}^{u^t},\quad \forall s\geq 3 & (\xi_{t,s})
\end{cases}$$

The utility function takes the standard CRRA form, $u(c)=\frac{c^{1-\rho}}{1-\rho}$, and we also assume $\beta=1/(1+r)$. This ensures that, absent any crisis, the optimal level of consumption would be constant across time. The first-order conditions yield the following Euler equations

$$\begin{aligned}
\varphi_t &=(1+r)(\Psi_t+\varphi_{t+1}) && (32)\\
\chi_t &=(1+r)\xi_{t,3} && (33)\\
\xi_{t,s} &=(1+r)\xi_{t,s+1},\quad \forall s\geq 3 && (34)\\
0 &=\sum_{t=0}^{\infty}[(f_t'(D)-r)\varphi_t+(f_{t+1}'(D)-r+\epsilon'(D))\Psi_t\\
&\quad +(f_{t+2}'(D)-r-(1+r)\epsilon'(D))\chi_t+\sum_{s=3}^{\infty}(f_{t+s}'-r)\xi_{t,s}] && (35)
\end{aligned}$$

We define two variables that summarize the depth of a balance of payment crisis.

$$0 < x = \frac{c^{u^t d}_{t+2} - c^{u^t d}_{t+1}}{c^{u^t d}_{t+1}} << 1 \tag{36}$$

$$0 < y = \frac{c^{u^{t+1}}_{t+1} - c^{u^t d}_{t+1}}{c^{u^t d}_{t+1}} << 1 \tag{37}$$

x captures the recovery in consumption following a crisis (measure 'across time'), whereas y is the percent difference in consumption across states of nature. x and y allow us to reformulate the following ratios of marginal utilities:

$$\frac{\Psi_t}{\varphi_{t+1}} = \frac{\pi}{1-\pi}\left(\frac{c^{u^t d}_{t+1}}{c^{u^{t+1}}_{t+1}}\right)^{-\rho} \approx \frac{\pi}{1-\pi}(1+\rho y) \tag{38}$$

$$\frac{\chi_t}{\Psi_t} = \beta\left(\frac{c^{u^t d}_{t+2}}{c^{u^t d}_{t+1}}\right)^{-\rho} \approx \beta(1-\rho x) \tag{39}$$

Note that in equation (39), although the coefficient of risk-aversion ρ appears in the equation, the appropriate concept is that of intertemporal elasticity of substitution. Substituting Ψ_t using equation (38) in equation (32), we find equation (40), and substituting equation (34) in equation (33), we find equation (41)

$$\varphi_t = (1+r)(1+\pi\rho y)\frac{\varphi_{t+1}}{1-\pi} \tag{40}$$

$$\xi_{t,s} = \frac{\chi_t}{(1+r)^{s-2}}\ , \quad \forall s \geq 3 \tag{41}$$

In addition, note that

$$\epsilon'(D)\Psi_t - (1+r)\epsilon'(D)\chi_t = \epsilon'(D)\Psi_t(1-(1+r)\beta(1-\rho x)) = \epsilon'(D)\rho x\Psi_t \tag{42}$$

which allows us to factor φ_t in equation (35) as

$$\begin{aligned} 0 = &\sum_{t=0}^{\infty}\varphi_t[(f'_t(D)-r) + (f'_{t+1}(D) - r + \rho x\epsilon'(D))\frac{\pi(1+\rho y)}{(1+r)(1+\pi\rho y)} \\ &+ (f'_{t+2}(D)-r)\frac{\pi(1+\rho y)(1-\rho x)}{(1+r)^2(1+\pi\rho y)} + \sum_{s=3}^{\infty}\frac{f'_{t+s}(D)-r}{(1+r)^t}\frac{(1-\rho x)(1+\rho y)\pi}{1+\pi\rho y}] \end{aligned} \tag{43}$$

Finally, we can factor φ_0 using equation (40) and $\varphi_0 = \varphi_t\frac{(1-\pi)^t}{(1+r)^t(1+\pi\rho y)^t}$

$$\begin{aligned} &\sum_{t=0}^{\infty}\frac{(f'_t(D)-r)(1-\pi)^t}{(1+r)^t(1+\pi\rho y)^t} + \sum_{t=1}^{\infty}\frac{(f'_t(D)-r+\epsilon'(D)\rho x)(1-\pi)^{t-1}\pi}{(1+r)^t(1+\pi\rho y)^t} \\ &+ \sum_{t=2}^{\infty}\frac{(f'_t(D)-r)(1-\pi)^{t-2}\pi}{(1+r)^t(1+\pi\rho y)^{t-1}\frac{1+\rho x}{1+\rho y}} + \sum_{j=3}^{\infty}\sum_{t\geq j}\frac{(f'_t(D)-r)(1-\pi)^{t-j}\pi}{(1+r)^t(1+\pi\rho y)^{t-j+1}\frac{1+\rho x}{1+\rho y}} = 0 \end{aligned} \tag{44}$$

which can be summarized as

$$\mathbb{E}_0\left[\sum_{t\geq 0}\frac{\tilde{f}'_t(D) - \tilde{m}^s_t}{1+\tilde{\theta}^s_t}\right] = 0 \tag{45}$$

where

$$\begin{cases} \tilde{m}_t^s = r & \text{if } s = u^t \\ \tilde{m}_t^s = r - \epsilon'(D)\rho x & \text{if } s = u^{t-1}d \end{cases}$$

$$\begin{cases} \tilde{\theta}_t^s = (1+r)^t(1+\pi\rho y)^t & \text{if } s = u^t, \text{ with prob.} = (1-\pi)^{t-1}\pi \\ \tilde{\theta}_t^s = (1+r)^t(1+\pi\rho y)^{t-1}\frac{1+\rho x}{1+\rho y} & \text{if } s = u^{t-1}d, \text{ with prob.} = (1-\pi)^{t-2}\pi \\ \tilde{\theta}_t^s = (1+r)^{t_0}(1+\pi\rho y)^{t_0-1}\frac{1+\rho x}{1+\rho y} & \text{if } s = u^{t_0-1}d^{1+t-t_0} \end{cases}$$

Infinite-horizon, recursive model

First-Order Conditions

Normal state
The first order conditions are

$$FOC : b_t \qquad u_c(c_t^U) + \beta\left\{(1-\pi)\frac{\partial V_{t+1}^U}{\partial b_t} + \pi\frac{\partial V_{t+1}^C}{\partial b_t}\right\} = 0 \tag{46}$$

$$FOC : R_t \qquad -u_c(c_t^U) + \beta\left\{(1-\pi)\frac{\partial V_{t+1}^U}{\partial R_t} + \pi\frac{\partial V_{t+1}^C}{\partial R_t}\right\} = 0 \tag{47}$$

$$FOC : K_t \qquad u_c(c_t^U)\left(-1-\chi(K_t-K_{t-1})\right) + \beta\left\{(1-\pi)\frac{\partial V_{t+1}^U}{\partial K_t} + \pi\frac{\partial V_{t+1}^C}{\partial K_t}\right\} = 0 \tag{48}$$

Crisis state

$$FOC : b_t \qquad \beta^{1/2}u_c(c_{t,2}^C) + \beta\left\{(1-\pi)\frac{\partial V_{t+1}^U}{\partial b_t} + \pi\frac{\partial V_{t+1}^C}{\partial b_t}\right\} = 0 \tag{49}$$

$$FOC : R_t \qquad -\beta^{1/2}u_c(c_{t,2}^C) + \beta\left\{(1-\pi)\frac{\partial V_{t+1}^U}{\partial R_t} + \pi\frac{\partial V_{t+1}^C}{\partial R_t}\right\} = 0 \tag{50}$$

$$FOC : K_t \qquad \beta^{1/2}u_c(c_{t,2}^C)\left(-1-\chi(K_t-K_{t-1})\right) + \beta\left\{(1-\pi)\frac{\partial V_{t+1}^U}{\partial K_t} + \pi\frac{\partial V_{t+1}^C}{\partial K_t}\right\} = 0 \tag{51}$$

Envelope Conditions

Normal state

$$Env : b_t \qquad \frac{\partial V_t^U}{\partial b_{t-1}} = -(1+r)u_c(c_t^U) \tag{52}$$

$$Env : R_t \qquad \frac{\partial V_t^U}{\partial R_{t-1}} = u_c(c_t^U) \tag{53}$$

$$Env : K_t \qquad \frac{\partial V_t^U}{\partial K_{t-1}} = u_c(c_t^U)\left[f_K^U(K_{t-1}) + 1 - \delta - \chi\frac{K_t}{K_{t-1}^2}\left(\frac{K_t}{K_{t-1}} - 1\right)\right] \tag{54}$$

Crisis state

$$Env : b_t \qquad \frac{\partial V_t^C}{\partial b_{t-1}} = -(1+r)\beta^{1/2}u_c(c_{t,2}^C) \tag{55}$$

$$Env : R_t \qquad \frac{\partial V_t^C}{\partial R_{t-1}} = u_c(c_{t,1}^C) \tag{56}$$

$$Env : K_t \qquad \frac{\partial V_t^C}{\partial K_{t-1}} = u_c(c_{t,1}^C)\left[f_{K,1}^C(K_{t-1}) + 1 - \delta - \chi\frac{K_t}{K_{t-1}^2}\left(\frac{K_t}{K_{t-1}} - 1\right)\right] + u_c(c_{t,2}^C)\left[f_{K,2}^C(K_{t-1}) + 1 - \delta - \chi\frac{K_t}{K_{t-1}^2}\left(\frac{K_t}{K_{t-1}} - 1\right)\right] \tag{57}$$